TD 9: BDDs

Exercise 1 (Some BDDs)**.** Draw the BDDs for the following functions, using the order of your choice on the variables $\{x_1, x_2, x_3\}$:

1. the majority function $m(x_1, x_2, x_3)$: its value is 1 iff the majority of the input bits are 1's,
2. the hidden weighted bit function $h(x_1, x_2, x_3)$: its value is that of variable x_s, where $s = \sum_{i=1}^{3} x_i$ and x_0 is defined as 0.

Exercise 2 (Symmetric Functions)**.** A *symmetric function* of n variables has the same value for all permutations of the same n tuple of arguments.

Show that a BDD for a symmetric function has at most $\frac{n(n+1)}{2} + 1$ nodes (when omitting the 0-node).

Exercise 3 (Algorithms)**.** Describe linear-time algorithms for the following problems:

1. Counting the number of solutions of a boolean function f represented by a BDD, i.e. of the number of valuations ν s.t. $\nu \models f$.
2. Given two BDDs for boolean functions f and g, finding a valuation in the symmetric difference of f and g (i.e. it can be a satisfying assignment for f and not g or vice versa).

Exercise 4 (An Upper Bound on the Size of BDDs)**.** For a given n, let $P_n = \{x_1, \ldots, x_n\}$, where $x_i \prec x_j$ iff $i < j$.

1. For $n = 4$, find a BDD with as many nodes as possible. For $n = 5$, how many nodes would the largest possible BDD have?

For any $n \geq 1$ and $1 \leq k \leq n$, let $N_{k,n}$ the maximal number of nodes labelled with x_k in a BDD over P_n.

2. Show that $N_{k,n} \leq 2^{k-1}$.
3. Given k and n, how many different boolean functions exist on the variables $x_{k+1} \cdots x_n$? Deduce another bound for $N_{k,n}$.
4. How many nodes does a BDD over P_6 have at most? Over P_7?