Transducers

Session 6: Linear Regular Functions
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1 Streaming String Transducers

Exercise 1 (Two-way Streaming String Transducers). We define the notion of two way streaming string transducer as a streaming string transducer with registers, where the head can move in both directions. Let f be a function from Σ^* to Γ^*. Prove that the following are equivalent:

1. f is computed by a streaming string transducer.
2. f is computed by a two-way streaming string transducer.

2 Commutativity

This section is inspired by the Ph.D. thesis of Douéneau-Tabot [Dou23], and particularly of its Chapter 5 called Polyregular functions with commutative outputs on page 123.

A function f is commutative when, for all $n \in \mathbb{N}$, for all permutation $\sigma \in \mathcal{S}_n$, for all $w \in \Sigma^n$, $f(w) = f(\sigma(w))$. A function has unary output when the output alphabet is $\{1\}$.

Exercise 2 (Deciding Commutativity). Let f be a function from Σ^* to Γ^* computed by a 2DFT. Is it decidable whether f is commutative?

Exercise 3 (Unary Output). Let f be a function from Σ^* to $\{1\}^*$. Prove that the following are equivalent:

1. f is computed by a 2DFT.
2. f is computed by a rational function.

This is Theorem 5.15 in the case $k = 1$ in Douéneau-Tabot [Dou23].

Exercise 4 (Rational Series). A rational series is a function from Σ^* to K where K is a semi-ring that is computed by a weighted automaton. We recall that a weighted automaton W is a finite (non-deterministic) automaton with weights in $\mathbb{R}$, whose semantics is defined as $W(w)$ is the sum over all accepting runs of W of the product of the weights along this path. Equivalently, a weighted automaton is defined in terms of monoids by a morphism $\mu\colon \Sigma^* \to \mathcal{M}_{n,n}(\mathbb{R})$ and a linear map $\lambda\colon \mathcal{M}_{n,n}(\mathbb{R}) \to \mathbb{R}$, such that $W(w) := \lambda(\mu(w))$.

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†https://www.mimuw.edu.pl/~bojan/2023-2024/przeksztalcenia-automatowe-transducers
‡https://aliaumel.github.io/transducer-exercices/

We define counting formulas as a generalisation of the correspondence between $\mathbb{MSO}$ and regular languages to the case of rational series. A counting formula is a formula $\varphi \in \mathbb{MSO}$ with first-order and second-order free variables, whose semantics is given by $\#\varphi(w) := |\{\nu \text{ valuation } \mid w, \nu \models \varphi(w)\}|$.

Prove that the following are equivalent:

1. f is computed by a weighted automaton with $\mathbb{N}$-output,
2. f is computed by a counting formula.

What happens if you restrict formulas to be in $\mathbb{FO}$?

References

[Dou23] Gaëtan Douéneau-Tabot. “Optimization of string transducers”. PhD thesis. Université Paris-Cité, 2023. URL: https://gdoueneau.github.io/pages/DOUENEAU-TABOT_Optimization_of_string_transducers_v2.pdf.

A Hints