Accurate Approximate Diagnosability of Stochastic Systems

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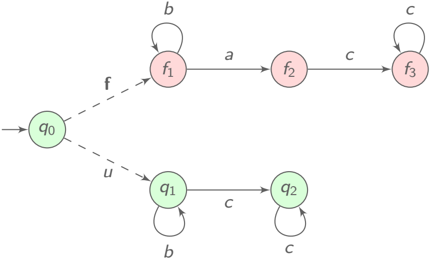
GdT Vasco-Mexico, March 9th 2016

Diagnosis Framework

LTS: Labelled transition system.

Diagnoser: must tell whether a fault **f** occurred, based on observations.

Convergence hypothesis: no infinite sequence of unobservable events.



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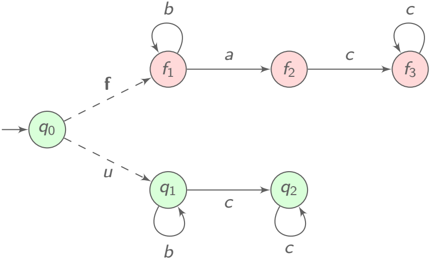

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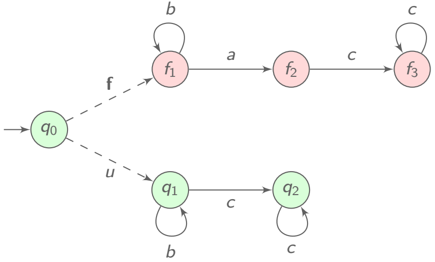

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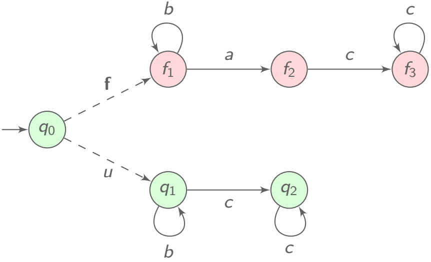

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? b is *ambiguous* as $\mathcal{P}^{-1}(b) = \{q_0 \xrightarrow{\mathbf{f}} f_1 \xrightarrow{b} f_1, q_0 \xrightarrow{u} q_1 \xrightarrow{b} q_1\}$.

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Diagnoser requirements:

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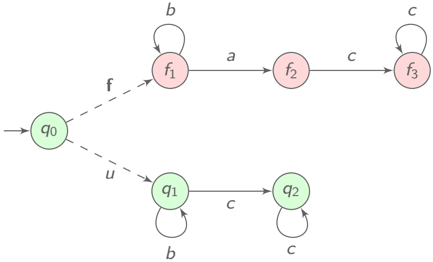
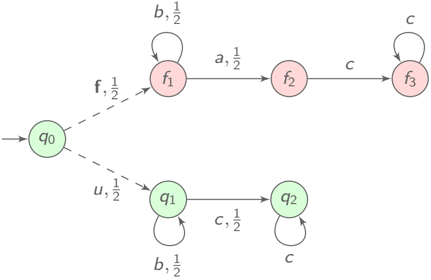


A sound but not reactive diagnoser : claiming a fault when a occurs.

Diagnosis of Probabilistic Systems [TT05]



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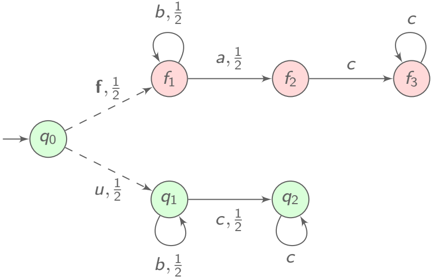

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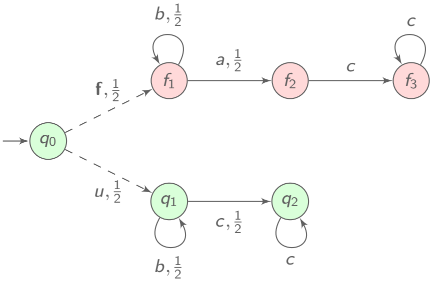

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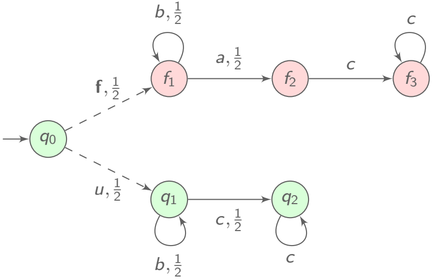

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How to adapt soundness and reactivity?

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Exact Diagnosis [BHL14]

An *exact diagnoser* fulfills

- Soundness: if a fault is claimed, a fault happened.

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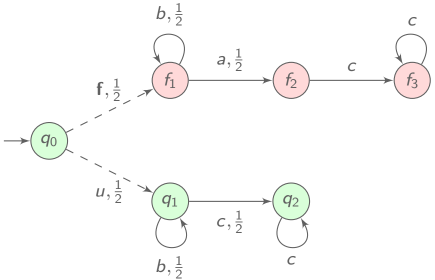

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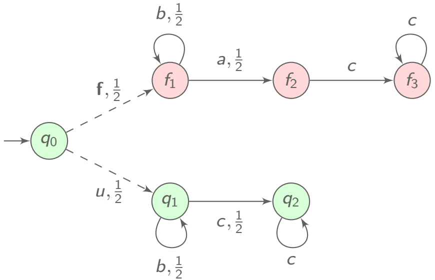

Exactly diagnosable.

Exact diagnosability is PSPACE-complete.

Also studied : exact prediction and prediagnosis.
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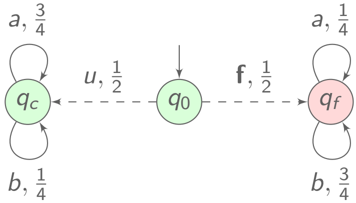
Exact Diagnosis versus Approximate Diagnosis



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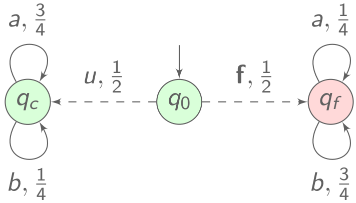

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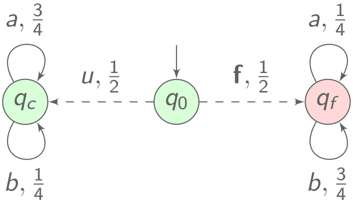

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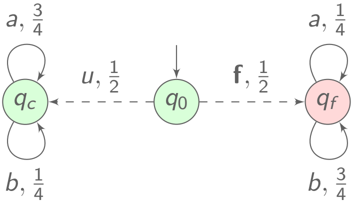

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However a high proportion of b implies a highly probable faulty run.

Relaxed Soundness: if a fault is claimed the probability of error is small.

Outline

Specification of Approximate Diagnosis

AA-diagnosis is Easy

Other Approximate Diagnoses are Hard

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Proportion of Correct Runs

Given an observation sequence $\sigma \in \Sigma_o^*$,

$$\mathsf{CorP}(\sigma) = \frac{\mathbb{P}(\{\pi^{-1}(\sigma) \cap \mathsf{correct}\})}{\mathbb{P}(\{\pi^{-1}(\sigma)\})}$$

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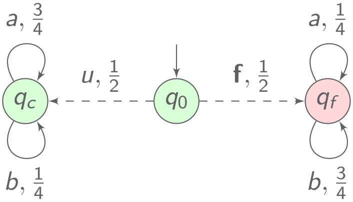

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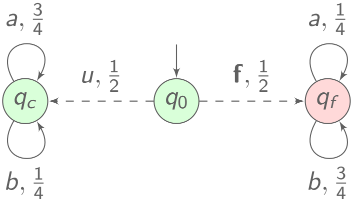

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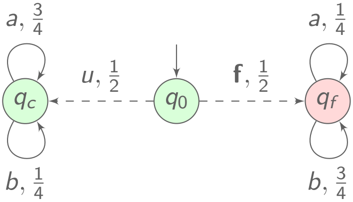

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0-diagnosers correspond to exact diagnosers.

Approximate Diagnosis Problems

	Reactivity	
Accuracy	*ε-diagnosability* Given $\varepsilon > 0$, does there exist an ε-diagnoser?	*uniform ε-diagnosability* Given $\varepsilon > 0$, does there exist a uniform ε-diagnoser?
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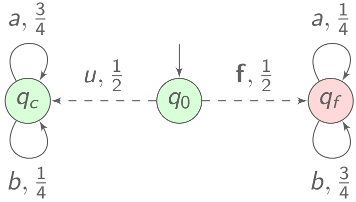
AA-diagnosability allows to select ε depending on external requirements.

Illustration



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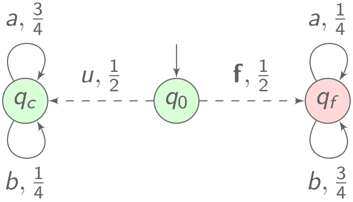

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Let $\rho = q_0 \xrightarrow{\mathbf{f}} q_f \cdots q_f$. Let ρ_f extending ρ.

Almost surely, when $|\rho_f| \to \infty$, ρ_f has more b's than a's.

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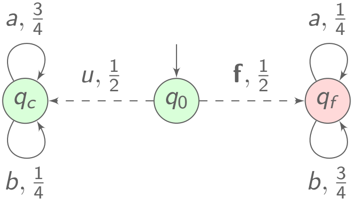

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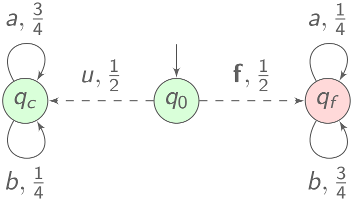

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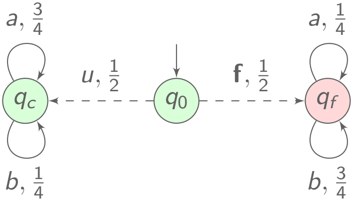

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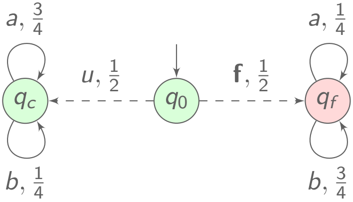

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$\mathbb{P}(\rho_f | \rho \preceq \rho_f \wedge |\rho_f| = k + |\rho| \wedge \mathrm{CorP}(\mathcal{P}(\rho_f)) \leq \varepsilon) \leq \alpha \mathbb{P}(\rho)$ implies $k \geq n$.

Establishing relations between the Specifications

$$
\begin{tikzcd}
\text{uniformly exact diagnosis} \arrow[r, Rightarrow] \arrow[d, Leftrightarrow] & \text{uniformly AA-diagnosable} \arrow[r, Rightarrow, "\text{for all } \varepsilon"] \arrow[d, Rightarrow] & \text{uniformly } \varepsilon\text{-diagnosable} \arrow[d, Rightarrow] \\
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Complexity of the Problems

	Simple	Uniform
ε-diagnosability	undecidable	undecidable
AA-diagnosability	PTIME	undecidable

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A Simple Case

Initial fault pLTS. Initially, an unobservable split towards two subpLTS:

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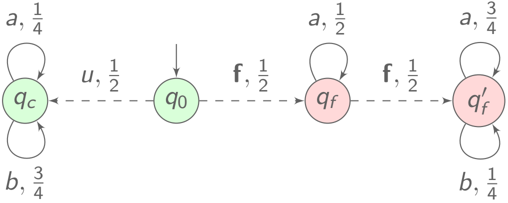
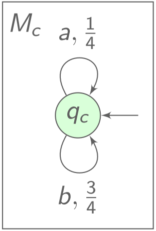
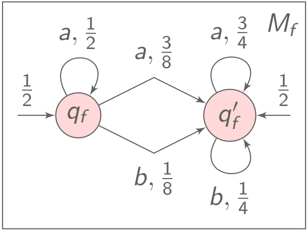


- an initial state, q_0;
- an arbitrary pLTS with states $\{q_f, q'_f\}$;
- a correct pLTS with state q_c.

Solving AA-diagnosability for Initial-Fault pLTS

- Transform the correct and arbitrary subpLTS in *labelled Markov chains* by merging the unobservable transitions.





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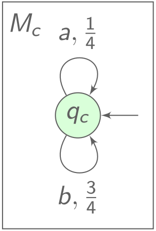

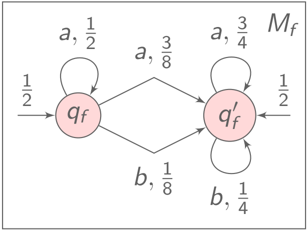

• $\mathbb{P}^M(E)$ = measure of infinite runs of M with observation in E.
Distance 1 problem: $\exists E$ (measurable) $\subseteq \Sigma_o^\omega, \mathbb{P}^{M_c}(E) - \mathbb{P}^{M_f}(E) = 1$?

• Illustration: $E = \{\sigma \mid \limsup_{n\to\infty} \frac{|\sigma_{\downarrow n}|_b}{|\sigma_{\downarrow n}|_a} > 1\}$

[CK14] Chen and Kiefer

On the Total Variation Distance of Labelled Markov Chains, CSL-LICS'14.

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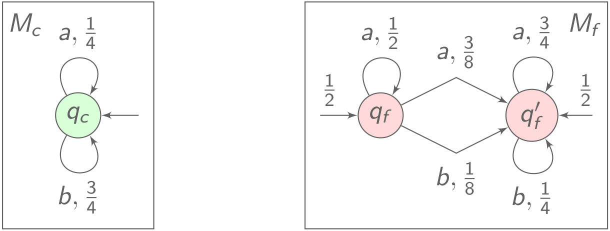

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The distance 1 problem is decidable in PTIME.

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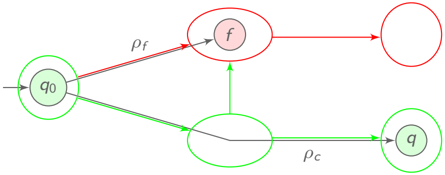

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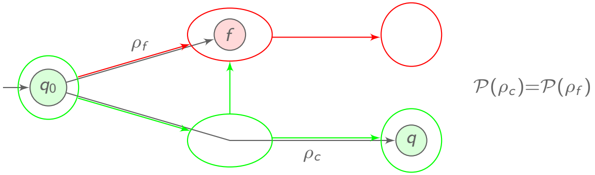
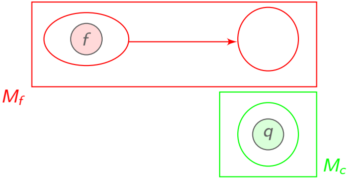


- Checking distance 1 for all relevant pairs.



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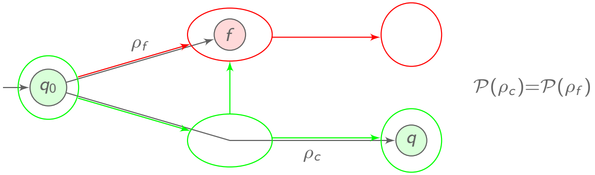

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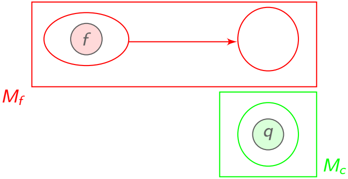

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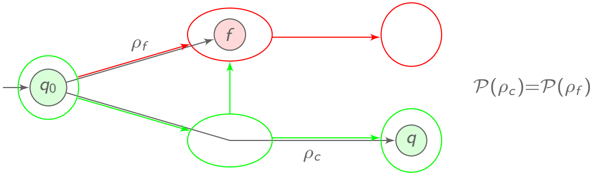
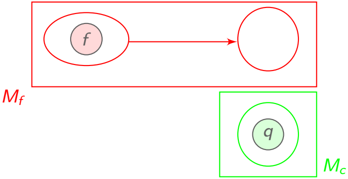


- Checking distance 1 for all relevant pairs.



AA-diagnosability is decidable in PTIME.

However an ε-diagnoser may need infinite memory.

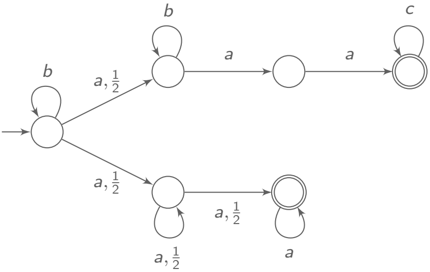
Outline

Specification of Approximate Diagnosis

AA-diagnosis is Easy

Other Approximate Diagnoses are Hard

The Emptiness Problem for Probabilistic Automata (PA)



$\mathbb{P}(b) = 0, \mathbb{P}(baa) = \frac{1}{4}, \mathbb{P}(baaa) = \frac{7}{8}$

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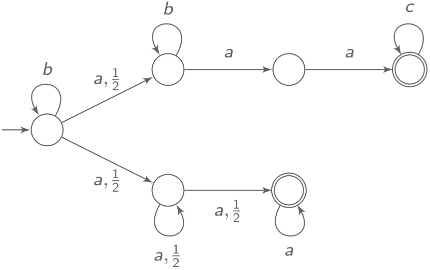

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Emptiness problem: Given a PA $\mathcal{A}$,
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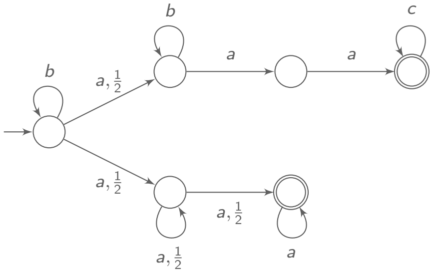

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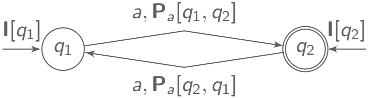
Emptiness problem: Given a PA $\mathcal{A}$,
$\exists w \in \Sigma^*, \mathbb{P}_{\mathcal{A}}(w) > \frac{1}{2}$?

The emptiness problem for PA is undecidable even when for all w,
$\frac{1}{4} \leq \mathbb{P}_{\mathcal{A}}(w) \leq \frac{3}{4}$.

[P71] Paz, *Introduction to Probabilistic Automata*, Academic Press 1971.

From PA to Uniform AA-diagnosability



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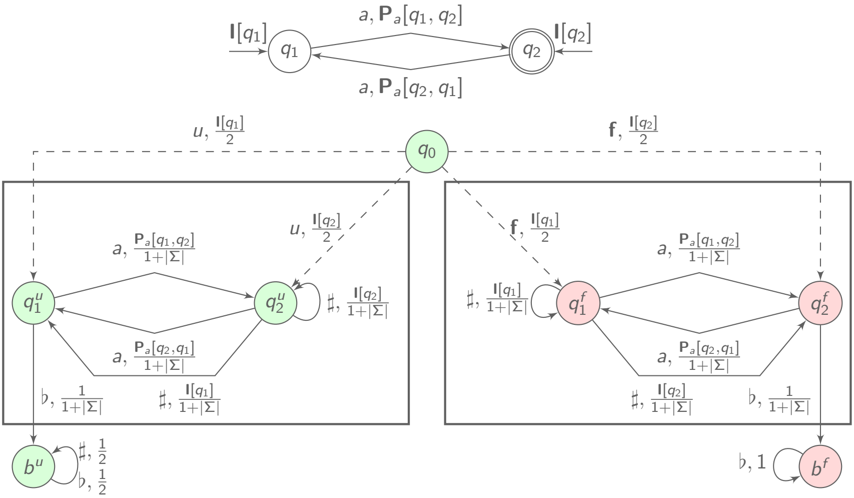

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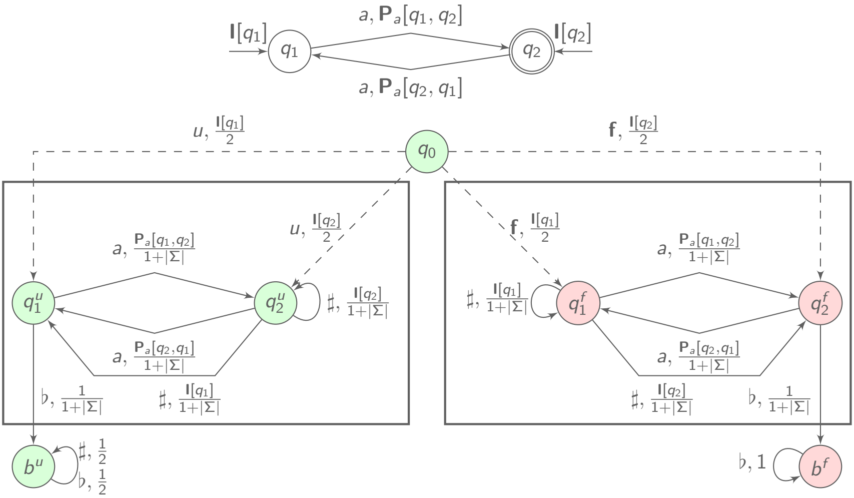

If $\exists w \in \Sigma_o^*, \mathbb{P}_{\mathcal{A}}(w) > 1/2$ then $\lim_{n \to \infty} \mathrm{CorP}((w\sharp)^n \flat) = 1$.

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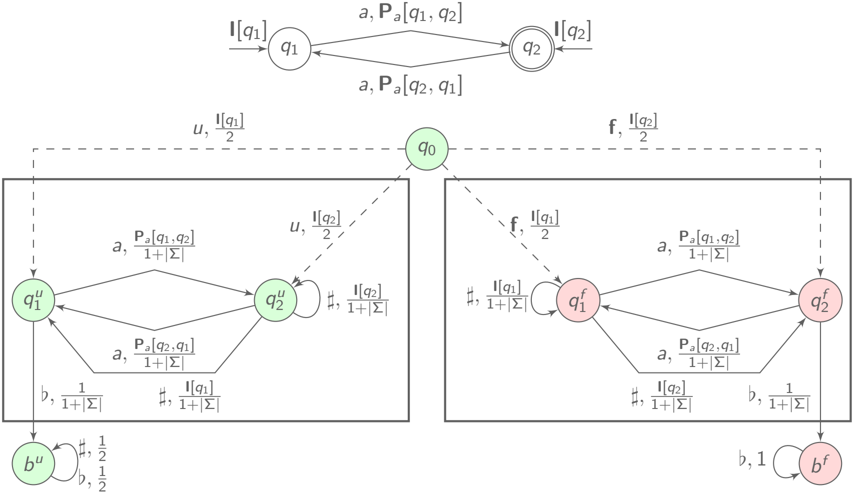

If $\exists w \in \Sigma_o^*, \mathbb{P}_{\mathcal{A}}(w) > 1/2$ then $\lim_{n \to \infty} \mathrm{CorP}((w\sharp)^n \flat) = 1$.

If $\forall w \in \Sigma_o^*, \mathbb{P}_{\mathcal{A}}(w) \leq 1/2$ then $\forall n\ \mathrm{CorP}((w\sharp)^n \flat) \leq \frac{3}{4}$.

Conclusion

Contributions

- Investigation of semantical issues
- Complexity of the notions of approximate diagnosis
  - A PTIME algorithm for AA-diagnosability
  - Undecidability of other approximate diagnosability

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Future work

- Approximate prediction and prediagnosis
- Diagnosis of infinite state stochastic systems