Outline

Expressivity

Definition: Equivalence

Let $\mathcal{C}$ be a class of time flows.

Two formulae $\varphi, \psi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ are equivalent over $\mathcal{C}$ if for all temporal structures $w = (\mathbb{T}, <, h)$ over $\mathcal{C}$ and all time points $t \in \mathbb{T}$ we have

$$w, t \models \varphi \quad \text{iff} \quad w, t \models \psi$$

Two formulae $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ and $\psi(x) \in \mathrm{FO_{AP}}(<)$ are equivalent over $\mathcal{C}$ if for all temporal structures $w = (\mathbb{T}, <, h)$ over $\mathcal{C}$ and all time points $t \in \mathbb{T}$ we have

$$w, t \models \varphi \quad \text{iff} \quad w, x \mapsto t \models \psi$$

We also write $w \models \psi(t)$.

Remark: $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS}) \subseteq \mathrm{FO}^3_{\mathrm{AP}}(<) \subseteq \mathrm{FO_{AP}}(<)$

$\forall \varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$, $\exists \psi(x) \in \mathrm{FO}^3_{\mathrm{AP}}(<)$ such that φ and $\psi(x)$ are equivalent.

Expressivity

Definition: complete linear time flows

A time flow $(\mathbb{T}, <)$ is linear if $<$ is a total strict order.

A linear time flow $(\mathbb{T}, <)$ is complete if every nonempty and bounded subset of $\mathbb{T}$ has a least upper bound and a greatest lower bound.

$(\mathbb{N}, <)$, $(\mathbb{Z}, <)$ and $(\mathbb{R}, <)$ are complete.
$(\mathbb{Q}, <)$ and $(\mathbb{R} \setminus \{0\}, <)$ are not complete.

Theorem: Expressive completeness [11, Kamp 68]

For complete linear time flows, $\quad \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS}) = \mathrm{FO_{AP}}(<)$

Elegant algebraic proof of $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}) = \mathrm{FO_{AP}}(<)$ over $(\mathbb{N}, <)$ due to Wilke 98.

See also Diekert-Gastin [17]: $\mathrm{TL} = \mathrm{FO} = \mathrm{SF} = \mathrm{AP} = \mathrm{CFBA} = \mathrm{VWAA}$.

Example:

$$\psi(x) = \neg P_a(x) \wedge \neg P_b(x) \wedge \forall y \forall z\, (P_a(y) \wedge P_b(z) \wedge y < z) \rightarrow \exists v\; y < v < z \wedge \begin{pmatrix} & P_c(v) \wedge x < y \\ \vee & P_d(v) \wedge z < x \\ \vee & P_e(v) \wedge y < x < z \end{pmatrix}$$

Stavi connectives: Time flows with gaps

Definition: Stavi Until: $\overline{\mathsf{U}}$

Let $w = (\mathbb{T}, <, h)$ be a temporal structure and $i \in \mathbb{T}$. Then, $w, i \models \varphi \,\overline{\mathsf{U}}\, \psi$ if

$$\begin{aligned} &\exists k\; i < k \\ &\wedge \exists j\, (i < j < k \wedge w, j \models \neg\varphi) \\ &\wedge \exists j\, (i < j < k \wedge \forall \ell\, (i < \ell < j \rightarrow w, \ell \models \varphi)) \\ &\wedge \forall j \left[i < j < k \rightarrow \left[\begin{array}{l} \exists k'\, [j < k' \wedge \forall j'\, (i < j' < k' \rightarrow w, j' \models \varphi)] \\ \vee\, [\forall \ell\, (j < \ell < k \rightarrow w, \ell \models \psi) \wedge \exists \ell\, (i < \ell < j \wedge w, \ell \models \neg\varphi)] \end{array} \right]\right] \end{aligned}$$

Similar definition for the Stavi Since $\overline{\mathsf{S}}$.

Example:

Let $w = (\mathbb{R} \setminus \{0\}, <, h)$ with $h(p) = \mathbb{R}_-$ and $h(q) = \mathbb{R}_+$.
Then, $w, -1 \not\models p \,\mathsf{SU}\, q$ but $w, -1 \models p \,\overline{\mathsf{U}}\, q$.

Theorem: [13, Gabbay, Hodkinson, Reynolds]

$\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS}, \overline{\mathsf{S}}, \overline{\mathsf{U}})$ is expressively complete for $\mathrm{FO_{AP}}(<)$ over the class of all linear time flows.

Stavi connectives: Time flows with gaps

Exercise: Isolated gaps

Let $\varphi_p = p \,\mathsf{SU}\, p \wedge \mathsf{SF}\, \neg p \wedge \neg(p \,\mathsf{SU}\, \neg p) \wedge \neg(p \,\mathsf{SU}\, \neg(p \,\mathsf{SU}\, \top))$.

Let $w = (\mathbb{T}, <, h)$ with $\mathbb{T} \subseteq \mathbb{R}$ and $t \in \mathbb{T}$.

Show that if $w, t \models \varphi_p$ then $\mathbb{T}$ has a gap.

Let $\psi_{p,q} = \varphi_p \wedge \ (q \vee \varphi_p) \,\mathsf{SU}\, (q \wedge \neg p)$.

Show that $\psi_{p,q}$ is equivalent to $p \,\overline{\mathsf{U}}\, q$ over the time flow $(\mathbb{R} \setminus \{0\}, <)$.

Show that $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ is $\mathrm{FO}_{\mathrm{AP}}(<)$-complete over the time flow $(\mathbb{R} \setminus \mathbb{Z}, <)$.

Temporal depth

Definition: Temporal depth of $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$

$$\begin{aligned}
\mathrm{td}(p) &= 0 && \text{if } p \in \mathrm{AP} \\
\mathrm{td}(\neg\varphi) &= \mathrm{td}(\varphi) \\
\mathrm{td}(\varphi \vee \psi) &= \max(\mathrm{td}(\varphi), \mathrm{td}(\psi)) \\
\mathrm{td}(\varphi \,\mathsf{SS}\, \psi) &= \max(\mathrm{td}(\varphi), \mathrm{td}(\psi)) + 1 \\
\mathrm{td}(\varphi \,\mathsf{SU}\, \psi) &= \max(\mathrm{td}(\varphi), \mathrm{td}(\psi)) + 1
\end{aligned}$$

Lemma:

Let $B \subseteq \mathrm{AP}$ be finite and $k \in \mathbb{N}$.
There are (up to equivalence) finitely many formulae in $\mathrm{TL}(B, \mathsf{SU}, \mathsf{SS})$ of temporal depth at most k.

k-equivalence

Definition:

Let $w_0 = (\mathbb{T}_0, <, h_0)$ and $w_1 = (\mathbb{T}_1, <, h_1)$ be two temporal structures.
Let $i_0 \in \mathbb{T}_0$ and $i_1 \in \mathbb{T}_1$. Let $k \in \mathbb{N}$.

We say that (w_0, i_0) and (w_1, i_1) are k-equivalent, denoted $(w_0, i_0) \equiv_k (w_1, i_1)$, if they satisfy the same formulae in $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ of temporal depth at most k.

Lemma: $\equiv_k$ is an equivalence relation of finite index.

Example:

Let $a = \{p\}$ and $b = \{q\}$. Let $w_0 = babaababaa$ and $w_1 = baababaaba$.

$$\begin{aligned}
&(w_0, 3) \equiv_0 (w_1, 4) \\
&(w_0, 3) \equiv_1 (w_1, 4)\ ? \\
&(w_0, 3) \equiv_1 (w_1, 6)\ ?
\end{aligned}$$

Here, $\mathbb{T}_0 = \mathbb{T}_1 = \{0, 1, 2, \ldots, 9\}$.

EF-games for $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$

The EF-game has two players: Spoiler (Player I) and Duplicator (Player II).

The game board consists of 2 temporal structures:
$w_0 = (\mathbb{T}_0, <, h_0)$ and $w_1 = (\mathbb{T}_1, <, h_1)$.

There are two tokens, one on each structure: $i_0 \in \mathbb{T}_0$ and $i_1 \in \mathbb{T}_1$.

A configuration is a tuple (w_0, i_0, w_1, i_1)
or simply (i_0, i_1) if the game board is understood.

Let $k \in \mathbb{N}$.

The k-round EF-game from a configuration proceeds with (at most) k moves.

There are 2 available moves for $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$: SU-move or SS-move (see below).

Spoiler chooses which move is played in each round.

Spoiler wins if

- Either duplicator cannot answer during a move (see below).
- Or a configuration such that $(w_0, i_0) \not\equiv_0 (w_1, i_1)$ is reached.

Otherwise, duplicator wins.

Strict Until and Since moves

Definition: SU-move

- Spoiler chooses $\varepsilon \in \{0,1\}$ and $k_\varepsilon \in \mathbb{T}_\varepsilon$ such that $i_\varepsilon < k_\varepsilon$.
- Duplicator chooses $k_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ such that $i_{1-\varepsilon} < k_{1-\varepsilon}$.
  Spoiler wins if there is no such $k_{1-\varepsilon}$.
  Either spoiler chooses (k_0, k_1) as next configuration of the EF-game,
  or the move continues as follows
- Spoiler chooses $j_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ with $i_{1-\varepsilon} < j_{1-\varepsilon} < k_{1-\varepsilon}$.
- Duplicator chooses $j_\varepsilon \in \mathbb{T}_\varepsilon$ with $i_\varepsilon < j_\varepsilon < k_\varepsilon$.
  Spoiler wins if there is no such j_ε.
  The next configuration is (j_0, j_1).

Similar definition for the SS-move.

Winning strategy

Definition: Winning strategy

Duplicator has a winning strategy in the k-round EF-game starting from (w_0, i_0, w_1, i_1) if he can win all plays starting from this configuration.
This is denoted by $(w_0, i_0) \sim_k (w_1, i_1)$.

Spoiler has a winning strategy in the k-round EF-game starting from (w_0, i_0, w_1, i_1) if she can win all plays starting from this configuration.

Example:

Let $a = \{p\}$, $b = \{q\}$, $c = \{r\}$. Let $w_0 = aaabbc$ and $w_1 = aababc$.

$$(w_0, 0) \sim_1 (w_1, 0)$$
$$(w_0, 0) \not\sim_2 (w_1, 0)$$

Here, $\mathbb{T}_0 = \mathbb{T}_1 = \{0, 1, 2, \ldots, 5\}$.

EF-games for $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$

Lemma: Determinacy

The k-round EF-game for $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ is determined:
For each initial configuration, either spoiler or duplicator has a winning strategy.

Theorem: Soundness and completeness of EF-games

For all $k \in \mathbb{N}$ and all configurations (w_0, i_0, w_1, i_1), we have

$$(w_0, i_0) \sim_k (w_1, i_1) \text{ iff } (w_0, i_0) \equiv_k (w_1, i_1)$$

Example:

Let $a = \{p\}$, $b = \{q\}$, $c = \{r\}$.

Then, $aaabbc, 0 \models p \,\mathsf{SU}\, (q \,\mathsf{SU}\, r)$ but $aababc, 0 \not\models p \,\mathsf{SU}\, (q \,\mathsf{SU}\, r)$.

$p \,\mathsf{SU}\, (q \,\mathsf{SU}\, r)$ cannot be expressed with a formula of temporal depth at most 1.

$p \,\mathsf{SU}\, (q \wedge \mathsf{X}\, q)$ cannot be expressed with a formula of temporal depth at most 1.

Exercise:

On finite linear time flows, "even length" cannot be expressed in $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$.

Moves for Strict Future and Past modalities

Definition: SF-move

- Spoiler chooses $\varepsilon \in \{0,1\}$ and $j_\varepsilon \in \mathbb{T}_\varepsilon$ such that $i_\varepsilon < j_\varepsilon$.
- Duplicator chooses $j_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ such that $i_{1-\varepsilon} < j_{1-\varepsilon}$.
  Spoiler wins if there is no such $j_{1-\varepsilon}$.
  The new configuration is (j_0, j_1).

Similar definition for the SP-move.

Example:

$p \,\mathsf{SU}\, q$ is not expressible in $\mathrm{TL}(\mathrm{AP}, \mathsf{SP}, \mathsf{SF})$ over linear flows of time.

Let $a = \emptyset$, $b = \{p\}$ and $c = \{q\}$.

Let $w_0 = (abc)^n a (abc)^n$ and $w_1 = (abc)^n (abc)^n$.

If $n > k$ then, starting from $(w_0, 3n, w_1, 3n)$, duplicator has a winning strategy in the k-round EF-game using SF-moves and SP-moves.

Moves for Next and Yesterday modalities

Notation: $i \lessdot j \stackrel{\text{def}}{=} i < j \wedge \neg\exists k\,(i < k < j)$.

Definition: X-move

- Spoiler chooses $\varepsilon \in \{0,1\}$ and $j_\varepsilon \in \mathbb{T}_\varepsilon$ such that $i_\varepsilon \lessdot j_\varepsilon$.
- Duplicator chooses $j_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ such that $i_{1-\varepsilon} \lessdot j_{1-\varepsilon}$.
  Spoiler wins if there is no such $j_{1-\varepsilon}$.
  The new configuration is (j_0, j_1).

Similar definition for the Y-move.

Exercise:

Show that $p\,\mathsf{SU}\,q$ is not expressible in $\mathrm{TL}(\mathrm{AP}, \mathsf{Y}, \mathsf{SP}, \mathsf{X}, \mathsf{SF})$ over linear time flows.

Non-strict Until and Since moves

Definition: U-move

- Spoiler chooses $\varepsilon \in \{0,1\}$ and $k_\varepsilon \in \mathbb{T}_\varepsilon$ such that $i_\varepsilon \leq k_\varepsilon$.
- Duplicator chooses $k_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ such that $i_{1-\varepsilon} \leq k_{1-\varepsilon}$.
  Either spoiler chooses (k_0, k_1) as new configuration of the EF-game,
  or the move continues as follows
- Spoiler chooses $j_{1-\varepsilon} \in \mathbb{T}_{1-\varepsilon}$ with $i_{1-\varepsilon} \leq j_{1-\varepsilon} < k_{1-\varepsilon}$.
- Duplicator chooses $j_\varepsilon \in \mathbb{T}_\varepsilon$ with $i_\varepsilon \leq j_\varepsilon < k_\varepsilon$.
  Spoiler wins if there is no such j_ε.
  The new configuration is (j_0, j_1).

- If duplicator chooses $k_{1-\varepsilon} = i_{1-\varepsilon}$ then the new configuration must be (k_0, k_1).
- If spoiler chooses $k_\varepsilon = i_\varepsilon$ then duplicator must choose $k_{1-\varepsilon} = i_{1-\varepsilon}$, otherwise he loses.

Similar definition for the S-move.

Exercise:

1. Show that SU is not expressible in $\mathrm{TL}(\mathrm{AP}, \mathsf{S}, \mathsf{U})$ over $(\mathbb{R}, <)$.
2. Show that SU is not expressible in $\mathrm{TL}(\mathrm{AP}, \mathsf{S}, \mathsf{U})$ over $(\mathbb{N}, <)$.

Semantic Separation

Definition:

Let $w = (\mathbb{T}, <, h)$ and $w' = (\mathbb{T}, <, h')$ be temporal structures over the same time flow, and let $t \in \mathbb{T}$ be a time point.

- w, w' agree on t if $\ell(t) = \ell'(t)$
- w, w' agree on the past of t if $\ell(s) = \ell'(s)$ for all $s < t$
- w, w' agree on the future of t if $\ell(s) = \ell'(s)$ for all $s > t$

Recall: $h \colon \mathrm{AP} \to 2^{\mathbb{T}}$ and we let $\ell(t) = \{p \in \mathrm{AP} \mid t \in h(p)\}$.

Definition: Pure formulae and separation

Let $\mathcal{C}$ be a class of time flows. A formula φ over some logic $\mathcal{L}$ is pure past (resp. pure present, pure future) over $\mathcal{C}$ if

$$w, t \models \varphi \quad \text{iff} \quad w', t \models \varphi$$

for all temporal structures $w = (\mathbb{T}, <, h)$ and $w' = (\mathbb{T}, <, h')$ over $\mathcal{C}$
and all time points $t \in \mathbb{T}$ such that

w, w' agree on the past of t (resp. on t, on the future of t).

A logic $\mathcal{L}$ is separable over a class $\mathcal{C}$ of time flows if each formula $\varphi \in \mathcal{L}$ is equivalent to some (finite) boolean combination of pure formulae.

Syntactic Separation

Definition: Syntactically pure formulae and separation

A formula $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ is

- syntactically pure present if it is a boolean combinations of formulae in AP,
- syntactically pure future if it is a boolean combinations of formulae of the form $\alpha\,\mathsf{SU}\,\beta$ where $\alpha, \beta \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU})$,
- syntactically pure past if it is a boolean combinations of formulae of the form $\alpha\,\mathsf{SS}\,\beta$ where $\alpha, \beta \in \mathrm{TL}(\mathrm{AP}, \mathsf{SS})$.
- syntactically separated if it is a boolean combinations of syntactically pure formulae.

Example:

The formulae $\varphi_1 = \mathsf{SF}(q \wedge \mathsf{SP}\,p)$ and $\varphi_2 = \mathsf{SF}(q \wedge \neg\,\mathsf{SP}\,\neg p)$ are not separated but there are equivalent syntactically separated formulae.

Remark: Syntax versus semantic

Every formula $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ which is syntactically pure present (resp. future, past) is also semantically pure present (resp. future, past).

Separation

Theorem: [8, Gabbay, Pnueli, Shelah & Stavi 80]

$\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ is syntactically separable over discrete and complete linear orders.

Definition: Discrete linear order

A linear time flow $(\mathbb{T}, <)$ is discrete if every non-maximal element has an immediate successor and every non-minimal element has an immediate predecessor.

- $(\mathbb{N}, <)$ is the unique (up to isomorphism) discrete and complete linear order with a first point and no last point.
- $(\mathbb{Z}, <)$ is the unique (up to isomorphism) discrete and complete linear order with no first point and no last point.
- Any discrete and complete linear order is isomorphic to a sub-flow of $(\mathbb{Z}, <)$.

Theorem: Gabbay, Reynolds, see [7]

$\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ is syntactically separable over $(\mathbb{R}, <)$.

Initial equivalence

Definition: Initial Equivalence

Let $\mathcal{C}$ be a class of time flows having a least element (denoted 0).
Two formulae $\varphi, \psi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ are initially equivalent over $\mathcal{C}$ if for all temporal structures $w = (\mathbb{T}, <, h)$ over $\mathcal{C}$ we have

$$w, 0 \models \varphi \quad \text{iff} \quad w, 0 \models \psi$$

Two formulae $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ and $\psi(x) \in \mathrm{FO}_{\mathrm{AP}}(<)$ are initially equivalent over $\mathcal{C}$ if for all temporal structures $w = (\mathbb{T}, <, h)$ over $\mathcal{C}$ we have

$$w, 0 \models \varphi \quad \text{iff} \quad w \models \psi(0)$$

Corollary: of the separation theorem

For each $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ there exists $\psi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU})$ such that φ and ψ are initially equivalent over $(\mathbb{N}, <)$.

Initial equivalence

Example: $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ versus $\mathrm{TL}(\mathrm{AP}, \mathsf{SU})$

$$\mathsf{G}(\mathrm{grant} \to (\neg\mathrm{grant}\ \mathsf{SS}\ \mathrm{request}))$$

is initially equivalent to

$$(\mathrm{request}\ \mathsf{R}\ \neg\mathrm{grant}) \wedge \mathsf{G}(\mathrm{grant} \to (\mathrm{request} \vee (\mathrm{request}\ \mathsf{SR}\ \neg\mathrm{grant})))$$

Theorem: (Laroussinie & Markey & Schnoebelen 2002)

$\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ may be exponentially more succinct than $\mathrm{TL}(\mathrm{AP}, \mathsf{SU})$ over $(\mathbb{N}, <)$.

Separation and Expressivity

Theorem: [12, Gabbay 89] (already stated by Gabbay in 81)

Let $\mathcal{C}$ be a class of linear time flows.

Let $\mathcal{L}$ be a temporal logic able to express SF and SP.

Then, $\mathcal{L}$ is separable over $\mathcal{C}$ iff it is expressively complete for $\mathrm{FO}_{\mathrm{AP}}(<)$ over $\mathcal{C}$.

Exercise: Checking semantically pure

Is the following problem decidable? If yes, what is his complexity?

Input: A formula $\varphi \in \mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$

Question: Is the formula φ *semantically* pure future?

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