# Model-Checking Timed Temporal Logics

Patricia Bouyer

LSV – CNRS & ENS de Cachan – France

Oxford University Computing Laboratory – UK

Based on joint works with Fabrice Chevalier, Nicolas Markey,
Joël Ouaknine and James Worrell

# Outline

# Model-checking

system:

property:


$\Rightarrow$

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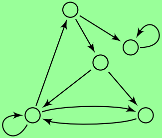

$\mathbf{G}\,(\texttt{request}\rightarrow\mathbf{F}\,\texttt{grant})$

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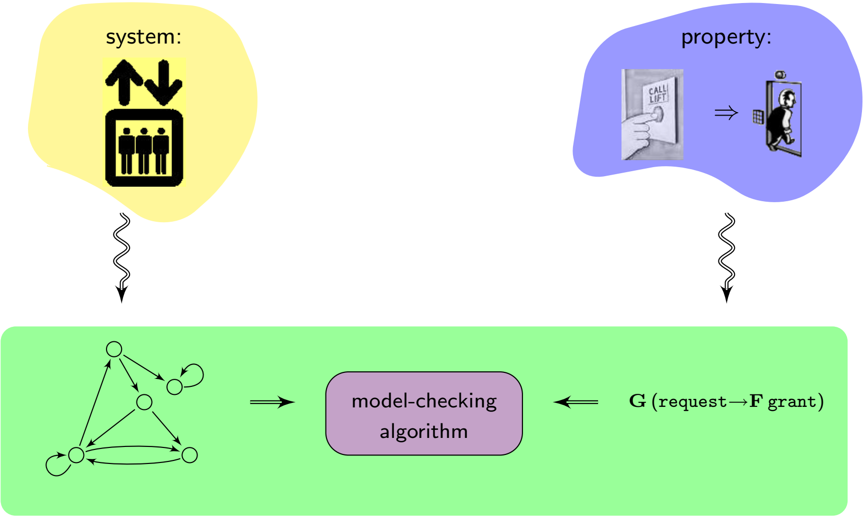

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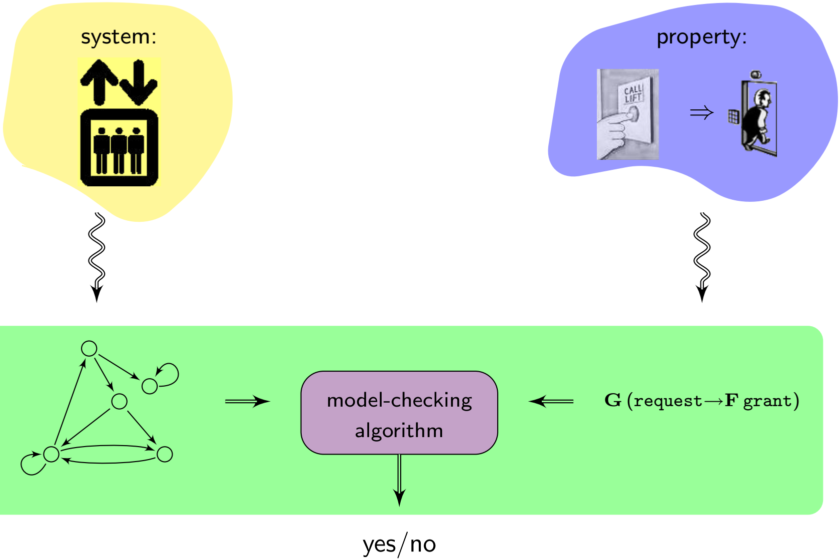

# The untimed (linear-time) framework

Linear-time temporal logic [Pnu77]

$$\mathsf{LTL} \ni \varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

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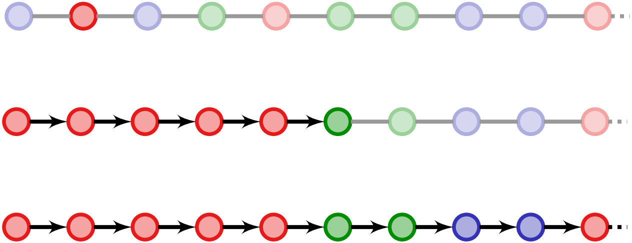

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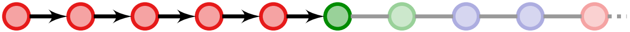 $\models \quad \bullet\,\mathbf{U}\,\bullet$

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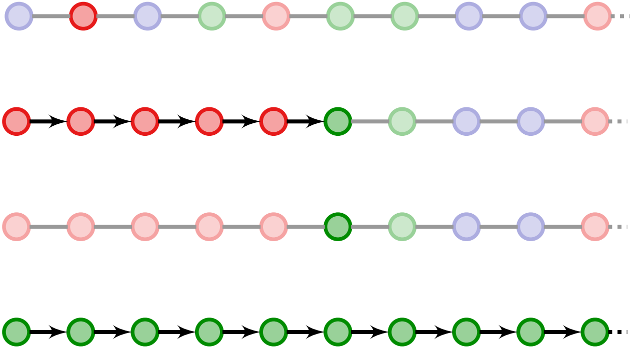

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- response property:

$$\mathbf{G}\,({\color{red}\bullet} \to \mathbf{F}\,{\color{green}\bullet})$$

- liveness property:

$$\mathbf{G}\,\mathbf{F}\,{\color{green}\bullet}$$

- safety property:

$$\mathbf{G}\,\neg{\color{red}\bullet}$$

- a more complex property:

$$({\color{green}\bullet} \wedge (\mathbf{F}\,{\color{blue}\bullet} \vee \mathbf{G}\,{\color{orange}\bullet}))\,\mathbf{U}\,{\color{red}\bullet}$$

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# Adding timing requirements

- Need for timed models
  - the behaviour of most systems depends on time;
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- Need for timed specification languages
  - the behaviour of most systems depends on time;
  - untimed specifications are not sufficient
    (for instance, bounded response timed, etc...)

☞ TCTL, MTL, TPTL, timed $\mu$-calculus...

# Outline

# Metric Temporal Logic (MTL)

[Koy90]

$$\mathsf{MTL} \ni \varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where $I$ is an interval with integral bounds.

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- Can be interpreted over timed words, or over signals
  - this distinction is fundamental

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- Can be interpreted over finite or infinite behaviours
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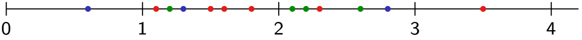
# The pointwise semantics

MTL formulas are interpreted over timed words:




$$(\bullet, .6)(\bullet, 1.1)(\bullet, 1.2)(\bullet, 1.3)\ldots$$

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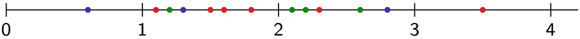


$$({\color{blue}\bullet}, .6)({\color{red}\bullet}, 1.1)({\color{green}\bullet}, 1.2)({\color{blue}\bullet}, 1.3)\dots$$

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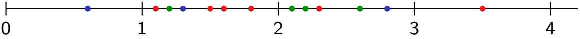


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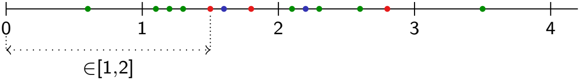


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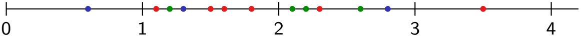


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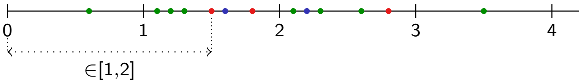


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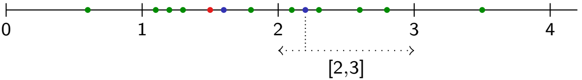


$\not\models\ \mathbf{G}_{[2,3]}\ \bullet$

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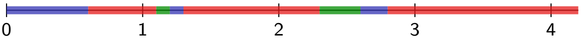
MTL formulas are interpreted over (finitely variable) signals:




$t \in [0,.6] \mapsto$ ■ $t \in (.6,1.1) \mapsto$ ■ $t \in [1.1,1.2) \mapsto$ ■

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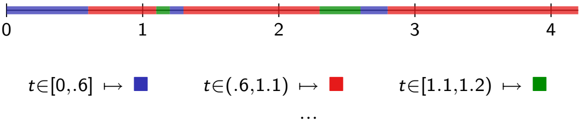


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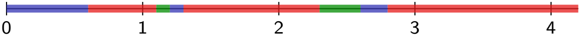


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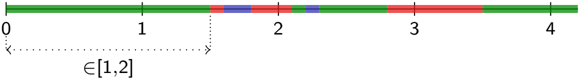


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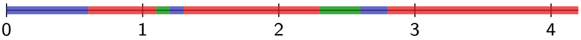


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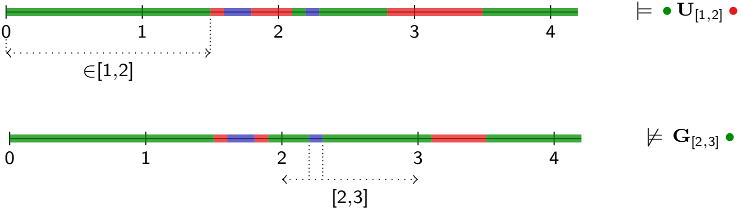

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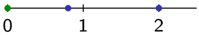


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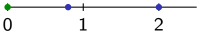


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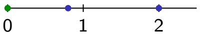


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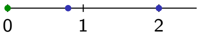


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  - in the pointwise semantics, $\mathbf{F}_{=2}\,\bullet \not\equiv \mathbf{F}_{=1}\,\mathbf{F}_{=1}\,\bullet$
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# Some further extensions

- Timed Propositional Temporal Logic (TPTL) [AH89]

TPTL = LTL + clock variables + clock constraints

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**Theorem**

LTL+Past is as expressive as LTL [Kam68,GPSS80].

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MTL is strictly less expressive than MTL+Past and TPTL [BCM05].

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**Conjecture in 1990:** the TPTL formula

$$\mathbf{G}\,(\bullet \to x.\mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

cannot be expressed in MTL.

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$$\mathbf{G}\,(\bullet \rightarrow x.\mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

cannot be expressed in MTL.

- This is true in the pointwise semantics.

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[BCM05] Bouyer, Chevalier, Markey. On the expressiveness of MTL and TPTL (FSTTCS'05).

# A note on the expressiveness

**Theorem**

LTL+Past is as expressive as LTL [Kam68,GPSS80].

**Theorem**

MTL is strictly less expressive than MTL+Past and TPTL [BCM05].

**Conjecture in 1990:** the TPTL formula

$$\mathbf{G}\,(\bullet \to x.\mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

cannot be expressed in MTL.

- This is true in the pointwise semantics.
- This is wrong in the continuous semantics!

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The TPTL formula

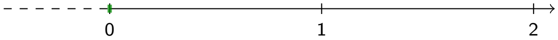
$$\mathbf{G}\left(\bullet \rightarrow x.\mathbf{F}\left(\bullet \wedge \mathbf{F}\left(\bullet \wedge x \leqslant 2\right)\right)\right)$$

can be expressed in MTL in the continuous semantics

The TPTL formula

$$\mathbf{G}\,(\bullet \rightarrow x.\mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

can be expressed in MTL in the continuous semantics

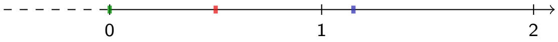



$$\mathbf{G}\ \bullet \ \rightarrow \ \left\{ \right.$$

The TPTL formula

$$\mathbf{G}\left({\color{green}\bullet} \to x.\mathbf{F}\left({\color{red}\bullet} \wedge \mathbf{F}\left({\color{blue}\bullet} \wedge x \leqslant 2\right)\right)\right)$$

can be expressed in MTL in the continuous semantics

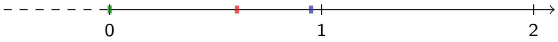



$$\mathbf{G}\ {\color{green}\bullet} \to \left\{ \begin{array}{l} \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet} \ \wedge\ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \\ \\ \end{array} \right.$$

The TPTL formula

$$\mathbf{G}\,(\textcolor{green}{\bullet} \rightarrow x.\mathbf{F}\,(\textcolor{red}{\bullet} \wedge \mathbf{F}\,(\textcolor{blue}{\bullet} \wedge x \leqslant 2)))$$

can be expressed in MTL in the continuous semantics




$$\mathbf{G}\ \textcolor{green}{\bullet} \ \rightarrow \ \left\{ \begin{array}{ll} & \mathbf{F}_{\leqslant 1}\ \textcolor{red}{\bullet} \ \wedge \ \mathbf{F}_{[1,2]}\ \textcolor{blue}{\bullet} \\ \vee & \\ & \mathbf{F}_{\leqslant 1}\left(\ \textcolor{red}{\bullet} \ \wedge \ \mathbf{F}_{\leqslant 1}\,\textcolor{blue}{\bullet}\ \right) \\ & \\ & \end{array} \right.$$

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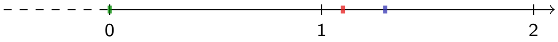


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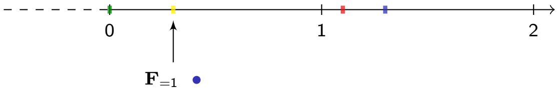


$$\mathbf{G}\ \textcolor{green}{\bullet}\ \rightarrow \begin{cases} \quad \mathbf{F}_{\leqslant 1}\ \textcolor{red}{\bullet}\ \wedge\ \mathbf{F}_{[1,2]}\ \textcolor{blue}{\bullet} \\ \vee \\ \quad \mathbf{F}_{\leqslant 1}\,(\ \textcolor{red}{\bullet}\ \wedge\ \mathbf{F}_{\leqslant 1}\,\textcolor{blue}{\bullet}\ ) \\ \vee \\ \end{cases}$$

The TPTL formula

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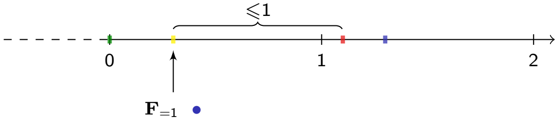


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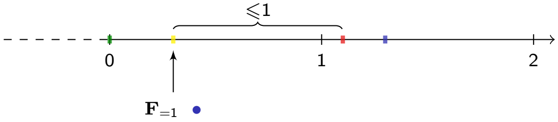


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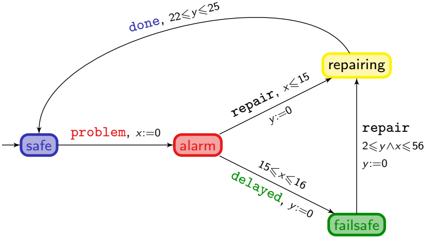
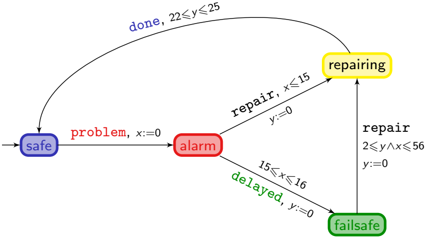
# Outline

# Timed automata

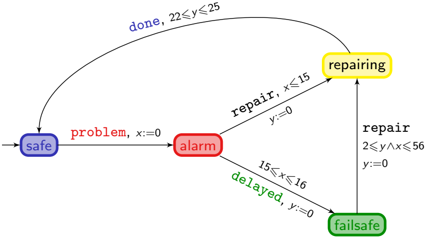


# Timed automata



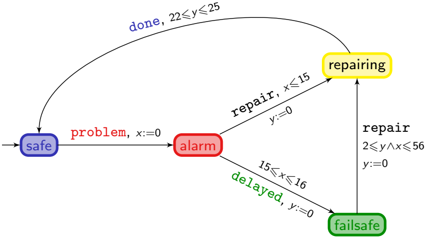

| | safe |
|---|---|
| x | 0 |
| y | 0 |

# Timed automata



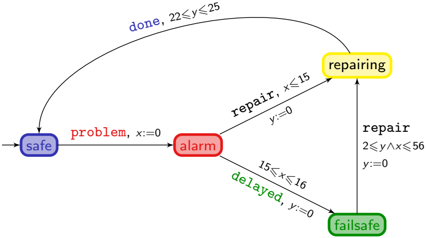

| | safe | $\xrightarrow{23}$ | safe |
|---|---|---|---|
| x | 0 | | 23 |
| y | 0 | | 23 |

# Timed automata



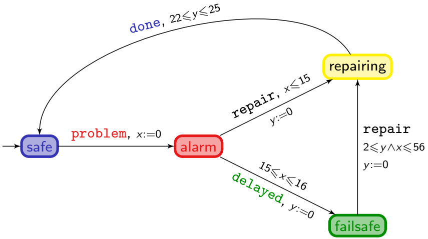

| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm |
|---|---|---|---|---|---|
| x | 0 | | 23 | | 0 |
| y | 0 | | 23 | | 23 |

# Timed automata



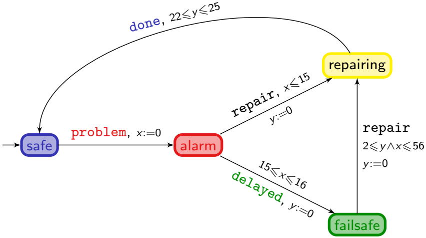

| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm |
|---|---|---|---|---|---|---|---|
| x | 0 | | 23 | | 0 | | 15.6 |
| y | 0 | | 23 | | 23 | | 38.6 |

# Timed automata



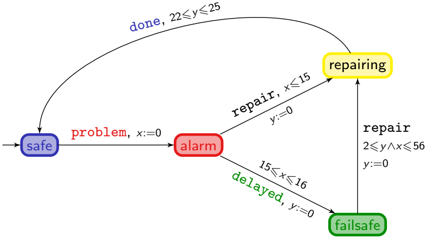

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe |
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 |

# Timed automata



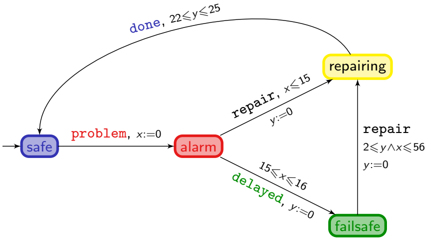

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe |
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 |

# Timed automata



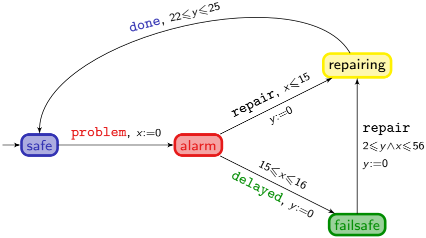

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe | $\xrightarrow{\texttt{repair}}$ | reparation |
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 | | 17.9 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 | | 0 |

# Timed automata



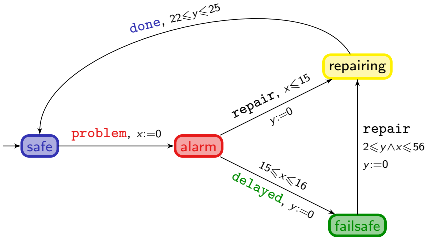

| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe | $\xrightarrow{\texttt{repair}}$ | reparation | $\xrightarrow{22.1}$ | reparation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 | | 17.9 | | 40 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 | | 0 | | 22.1 |

# Timed automata




| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe | $\xrightarrow{\texttt{repair}}$ | reparation | $\xrightarrow{22.1}$ | reparation | $\xrightarrow{\texttt{done}}$ | safe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 | | 17.9 | | 40 | | 40 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 | | 0 | | 22.1 | | 22.1 |

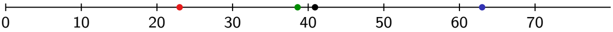
# Timed automata



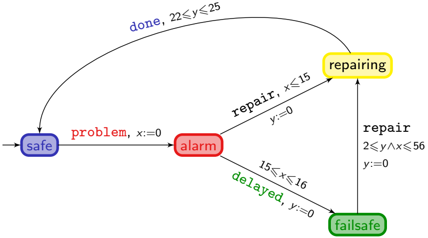

| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe | $\xrightarrow{\texttt{repair}}$ | reparation | $\xrightarrow{22.1}$ | reparation | $\xrightarrow{\texttt{done}}$ | safe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 | | 17.9 | | 40 | | 40 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 | | 0 | | 22.1 | | 22.1 |

Can be viewed:

- as the timed word (`problem`,23)(`delayed`,38.6)(`repair`,40.9)(`done`,63)



# Timed automata




| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | safe | $\xrightarrow{23}$ | safe | $\xrightarrow{\texttt{problem}}$ | alarm | $\xrightarrow{15.6}$ | alarm | $\xrightarrow{\texttt{delayed}}$ | failsafe | $\xrightarrow{2.3}$ | failsafe | $\xrightarrow{\texttt{repair}}$ | reparation | $\xrightarrow{22.1}$ | reparation | $\xrightarrow{\texttt{done}}$ safe |
| x | 0 | | 23 | | 0 | | 15.6 | | 15.6 | | 17.9 | | 17.9 | | 40 | 40 |
| y | 0 | | 23 | | 23 | | 38.6 | | 0 | | 2.3 | | 0 | | 22.1 | 22.1 |

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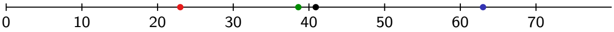
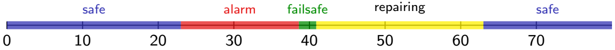



- as the signal



# Basic result on timed automata

**Theorem**

The reachability problem is decidable (and PSPACE-complete) for timed automata [AD94].

[AD94] Alur, Dill. A theory of timed automata (TCS, 1994).

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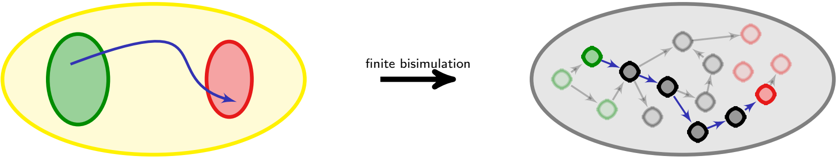


timed automaton

large (but finite) automaton
(region automaton)

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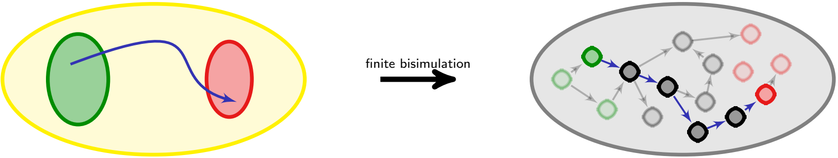


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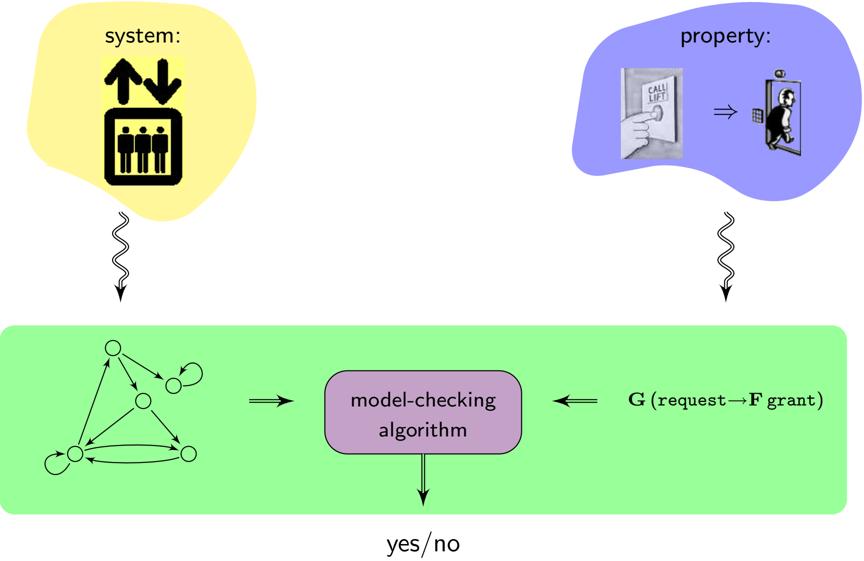
☞ It can be extended to model-check TCTL [ACD93].

[AD94] Alur, Dill. A theory of timed automata (TCS, 1994).
[ACD93] Alur, Courcoubetis, Dill. Model-checking in dense real-time (I&C, 1993).

# Outline

# Back to the model-checking problem

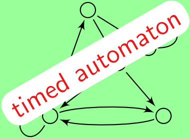


# Back to the model-checking problem

system:

property:

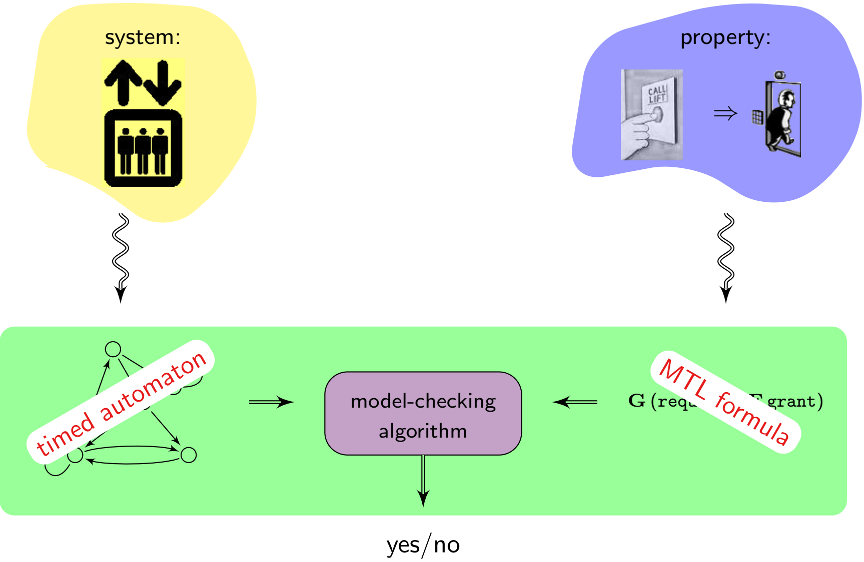

$\Rightarrow$




$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{G}\,(\texttt{request} \rightarrow \mathbf{F}\,\texttt{grant})$

yes/no

# Back to the model-checking problem



# Results

**Theorem**

Over finite runs, the model-checking problem is:

| | pointwise sem. | continuous sem. |
|---|---|---|
| MTL | decidable, NPR [OW05] | undecidable [AFH96] |
| MTL+Past | undecidable | undecidable |
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☞ we will explain this high complexity, following [Che07]

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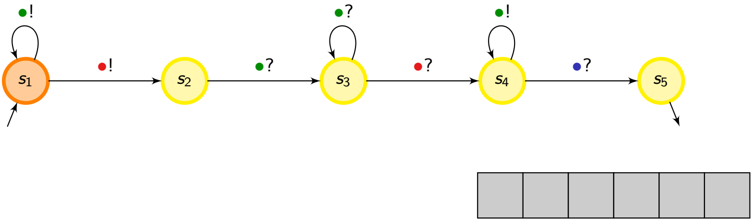
[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (Journal of the ACM, 1996).
[AH94] Alur, Henzinger. A really temporal logic (Journal of the ACM, 1994).
[Che07] Chevalier. Logiques pour les systèmes temporisés : contrôle et expressivité (PhD Thesis ENS Cachan, June 2007).

# A short visit to channel machines (1)

A channel machine = a finite automaton + a FIFO channel



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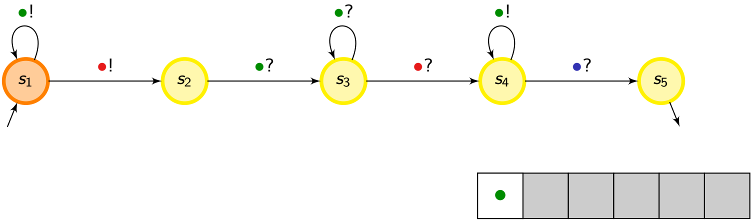

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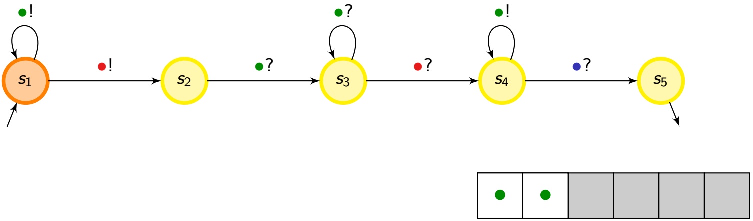

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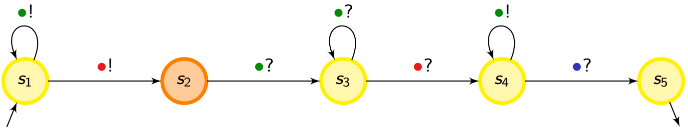


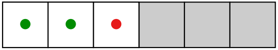

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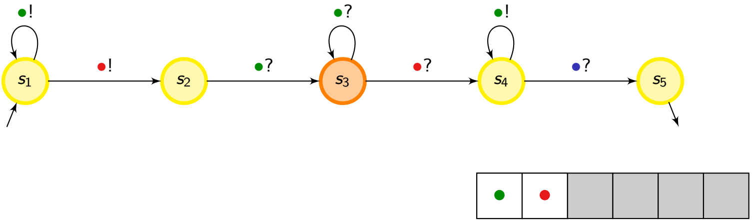

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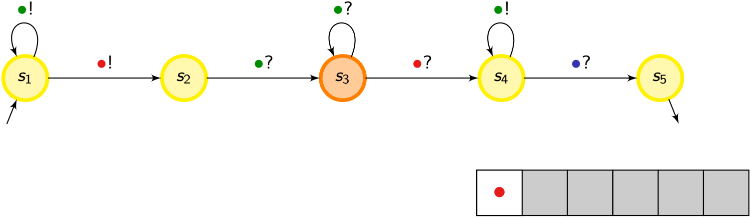

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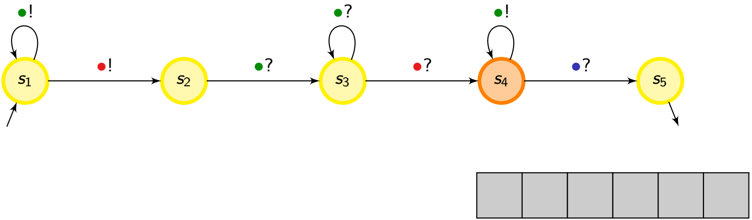

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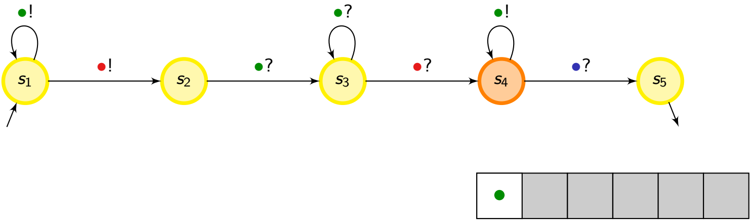

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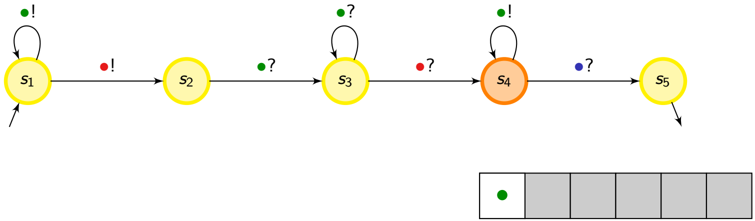


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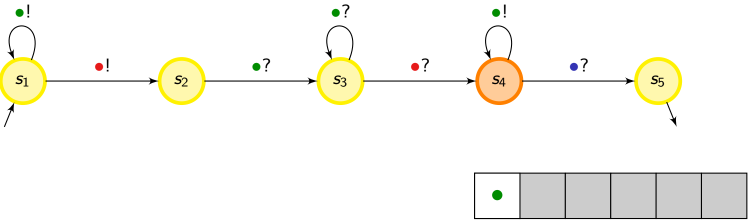


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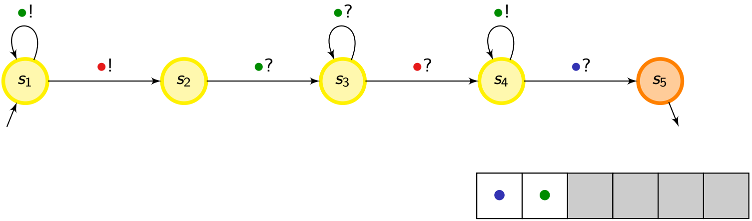


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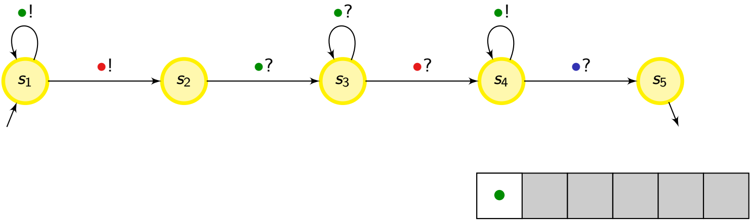


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**Proposition**

- The halting problem is undecidable for channel machines [BZ83].
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[BZ83] Brand, Zafiropulo. On communicating finite-state machines (Journal of the ACM, 1983).
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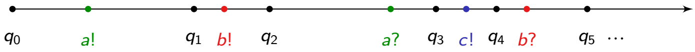
# Channel machines and timed words

We encode an execution of a channel machine as a timed word:

$$(q_0, \varepsilon) \xrightarrow{a!} (q_1, a) \xrightarrow{b!} (q_2, ab) \xrightarrow{a?} (q_3, b) \xrightarrow{c!} (q_4, bc) \xrightarrow{b?} (q_5, c) \cdots$$



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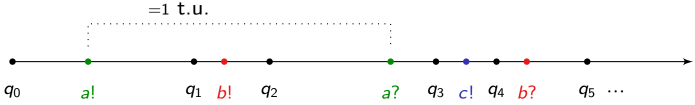

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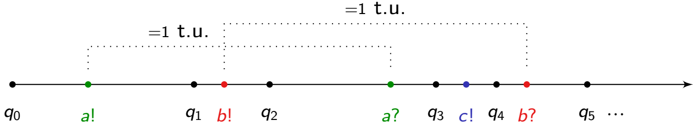

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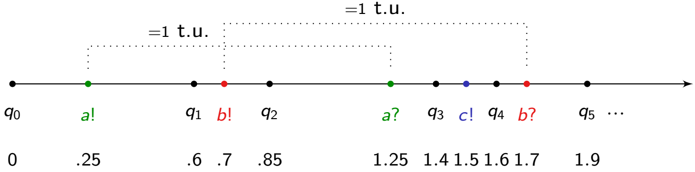


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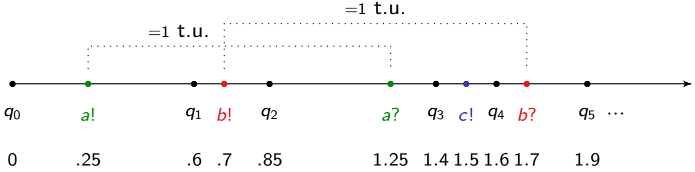


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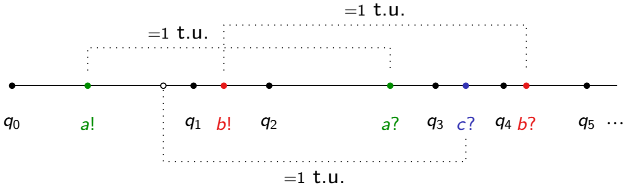

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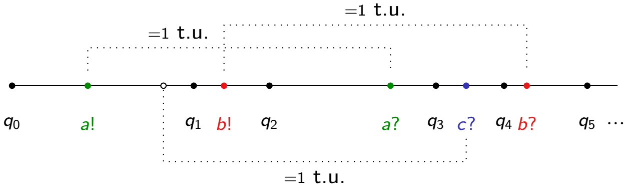


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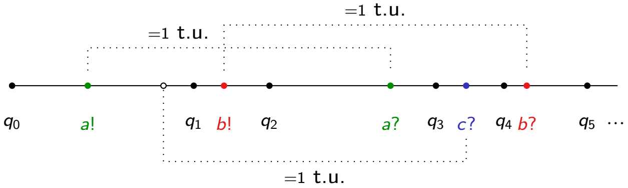


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☞ model-checking MTL is NPR

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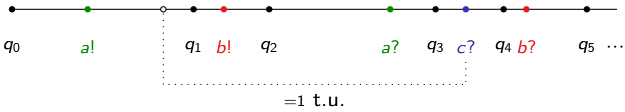

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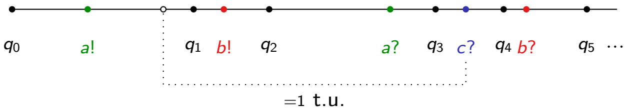


  - this formula is in TPTL (pointwise sem.), not in MTL

# What we have proved so far

**Theorem**

Over finite runs, the model-checking problem is:

| | pointwise sem. | continuous sem. |
|---|---|---|
| MTL | NPR [OW07] | undecidable [AFH96] |
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# What remains to be proved

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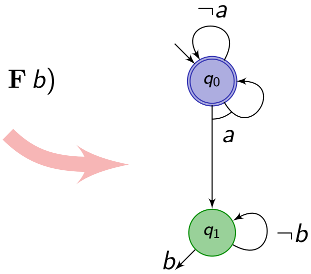

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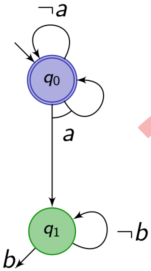


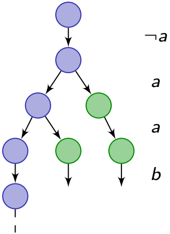

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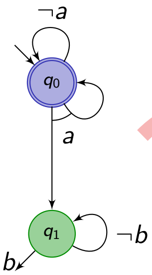


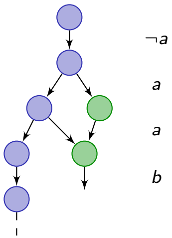

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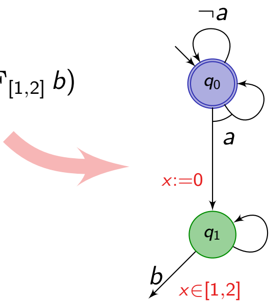

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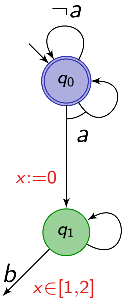


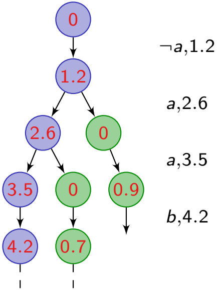

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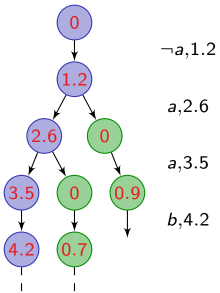

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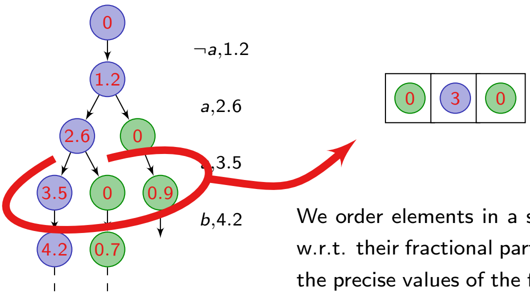


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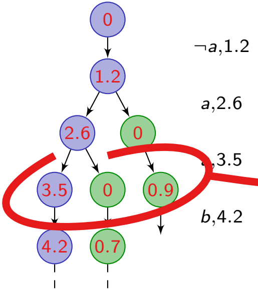


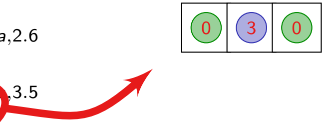


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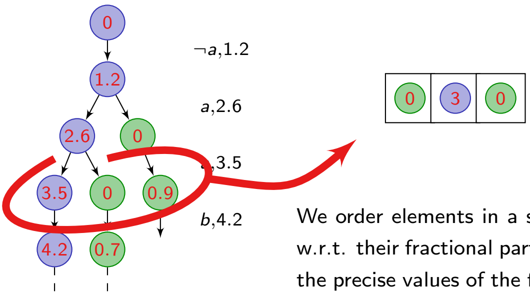


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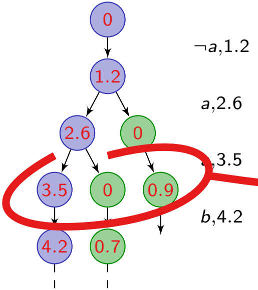


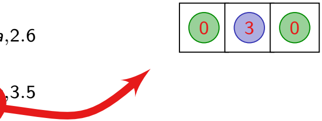


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- ☞ there is a well quasi-order on the set of abstract configurations (subword relation):

$$\text{higman} \sqsubseteq \text{highmountain}$$

# Summary

## Theorem

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$^\star$ by reduction of the recurrence problem for channel machines

[MH84] Miyano, Hayashi. Alternating finite automata on $\omega$-words (TCS, 1984).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

# Outline

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- Examples:
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Metric Interval Temporal Logic (MITL): [AFH96]

$$\mathsf{MITL} \ni \varphi \ ::= \ a \ \mid \ \neg\varphi \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \varphi\,\mathbf{U}_{I}\,\varphi$$

with $I$ a non-punctual interval

- Examples:
  - $\mathbf{G}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is not in MITL
  - $\mathbf{G}\,(\bullet \rightarrow \mathbf{F}_{[1,2]}\,\bullet)$ is in MITL

[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (Journal of the ACM, 1996).

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The model-checking problem for MITL is EXPSPACE-complete [AFH96].
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☞ something more clever needs to be done

[HR04] Hirshfeld, Rabinovich. Logics for real time: decidability and complexity (Fundamenta Informaticae, 2004).

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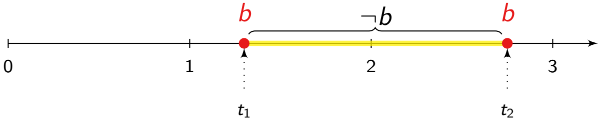

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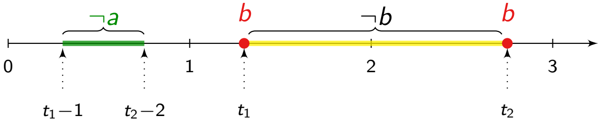

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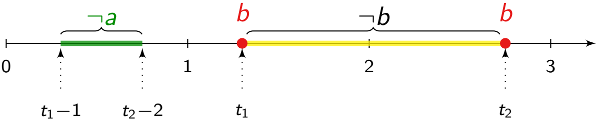


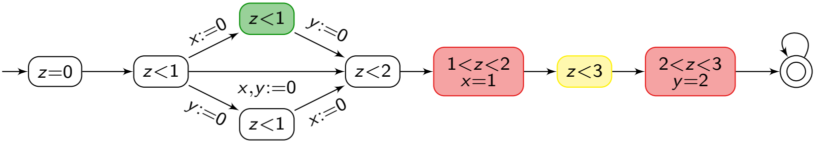

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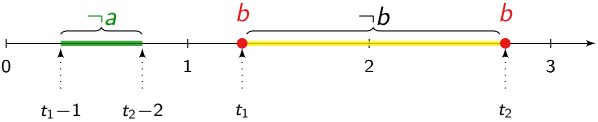


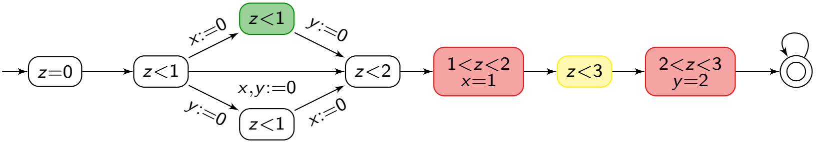


☞ This idea can be extended to any formula in MITL

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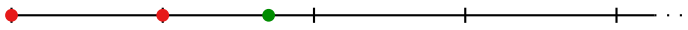

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- A Bounded-MTL formula may define a non timed-regular language:

$$\mathbf{G}_{\leqslant 1}\,(\textcolor{red}{\bullet} \rightarrow \mathbf{F}_{=1}\,\textcolor{green}{\bullet}) \wedge \mathbf{G}_{\leqslant 1}\,\textcolor{red}{\bullet} \wedge \mathbf{G}_{(1,2]}\,\textcolor{green}{\bullet}$$

defines the context-free language $\{\textcolor{red}{\bullet}^n \textcolor{green}{\bullet}^m \mid n \leqslant m\}$.

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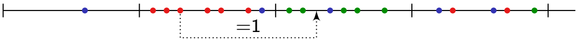

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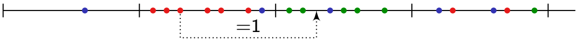


Offline, we stack all ‘relevant’ time units and use a sliding window:

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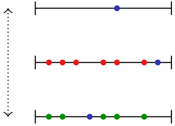

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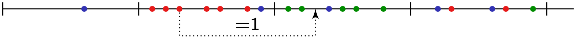


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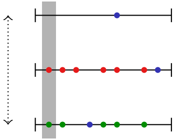

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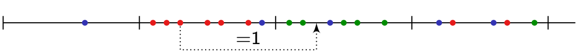


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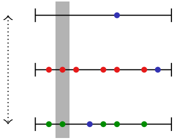

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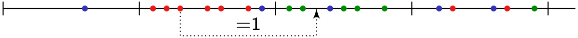


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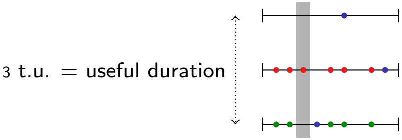

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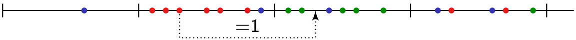


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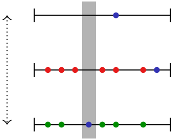

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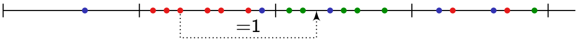


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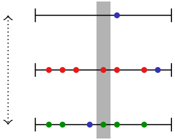

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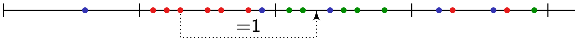


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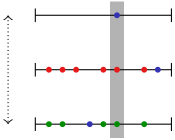

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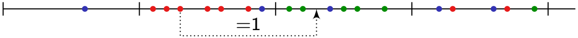


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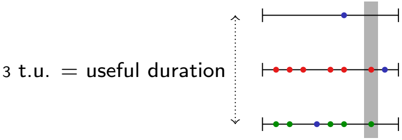

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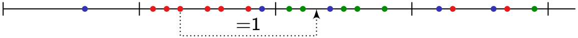


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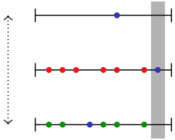

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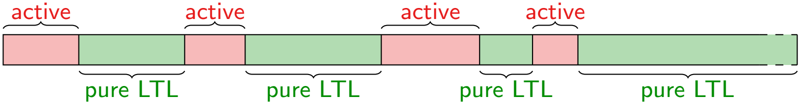


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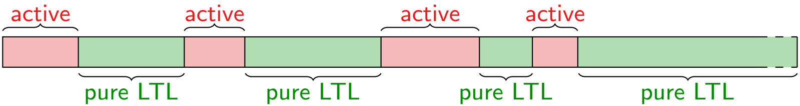


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- active fragment = cycle-bounded computation in a channel machine
- pure LTL part = finite automaton computation

# Outline

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- Recent advances have raised a new interest for linear-time timed temporal logics
  - Not everything is undecidable
  - Some rather 'efficient' subclasses
    - non-punctual formulas
    - structurally (co-)flat formulas
- A recent result: $\text{coFlat-MTL}_{\text{MITL}}$ unifies coFlat-MTL and MITL, and is EXPSPACE-complete [BMOW08]!
- No real data structures do exist for these logics.
- An interesting phenomenon (in the continuous semantics):

| | TCTL | MTL | Bounded-TCTL | Bounded-MTL |
|---|---|---|---|---|
| m.-c. | PSPACE | NPR/undec. | PSPACE | PSPACE |
| sat. | undec. | NPR/undec. | non-elem.⋆ | PSPACE |

⋆ ongoing work with Jenkins, Ouaknine, Worrell.

[BMOW08] Bouyer, Markey, Ouaknine, Worrell. A small-model theorem for real-time logics.