De l'analyse automatique de systèmes temporisés au contrôle de systèmes dynamiques

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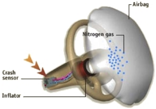
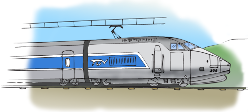
Time-dependent systems

- We are interested in timed systems





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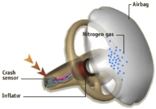
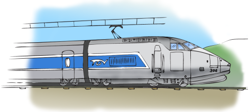
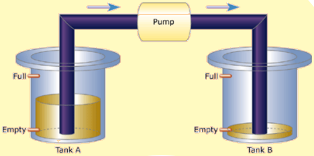




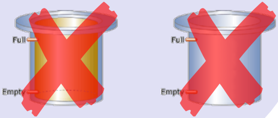
- ... and in their analysis and control

Model-checking and control

system:

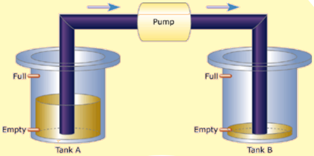


property:



Model-checking and control

system:



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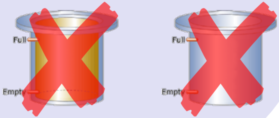
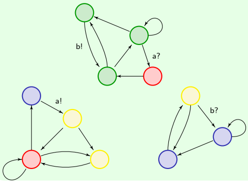


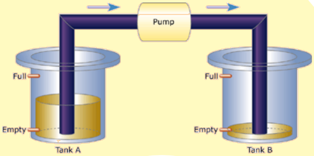


$\mathbf{AG}(\neg\texttt{B.overfull}$

$\quad\wedge\ \neg\texttt{B.dried\_up})$

Model-checking and control

system:



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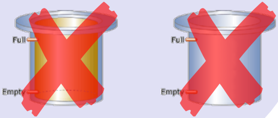
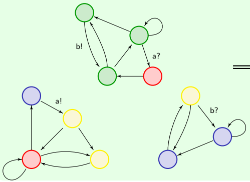




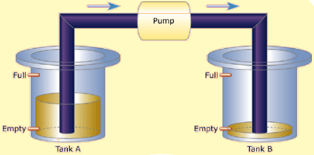
$\Longrightarrow$ algorithm $\Longleftarrow$

AG($\neg$`B.overfull`

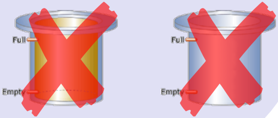
$\wedge$ $\neg$`B.dried_up`)

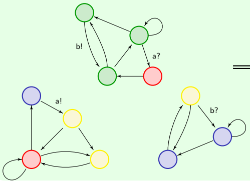
Model-checking and control

system:



property:



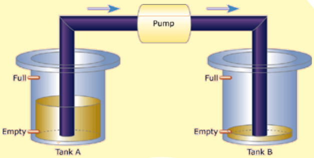


$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ **AG**$($¬`B.overfull` $\wedge$ ¬`B.dried_up`$)$

yes/no

Model-checking and control

system:



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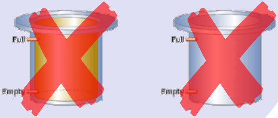
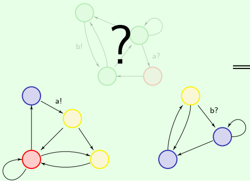
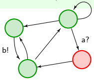




control/synthesis algorithm

$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$



An example: The task graph scheduling problem

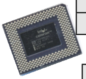
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

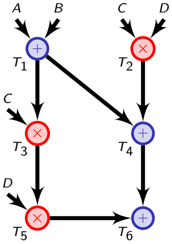
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata *(Communication of the ACM)*.

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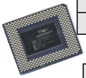

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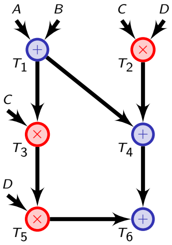

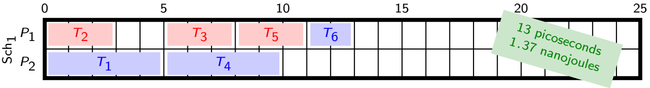

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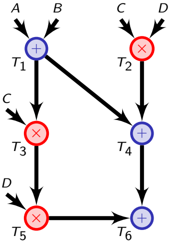

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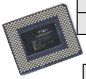
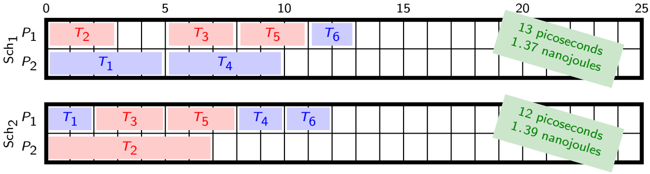


time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



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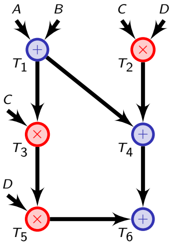

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time	
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×	3 picoseconds

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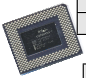
P_2 (slow):



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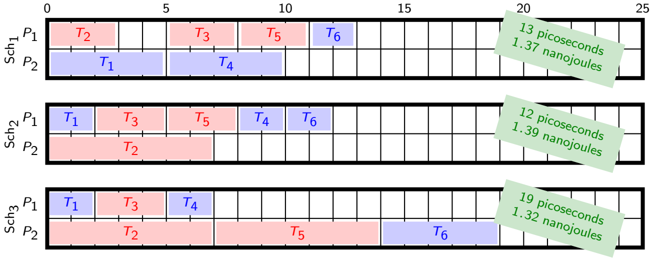

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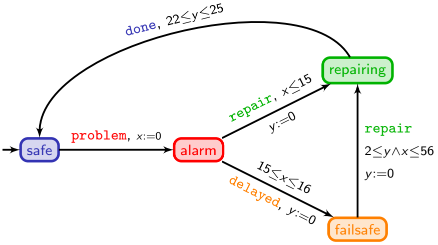
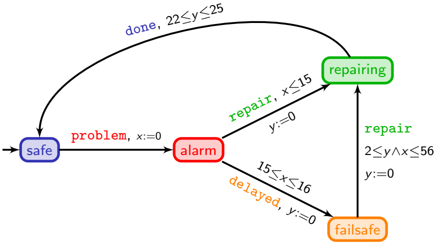
Outline

The model of timed automata



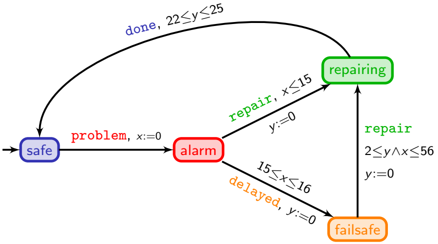
[AD94] Alur, Dill. A Theory of Timed Automata *(Theoretical Computer Science)*.

The model of timed automata



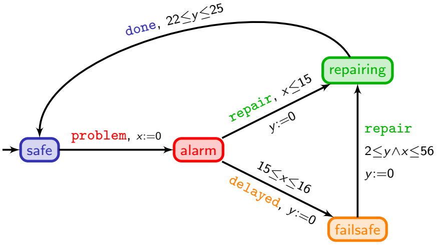
	safe
x	0
y	0

The model of timed automata



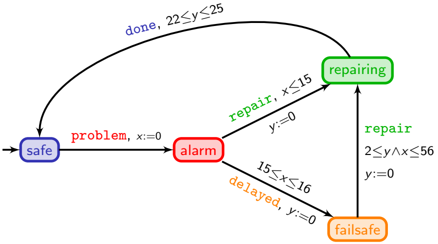
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

The model of timed automata



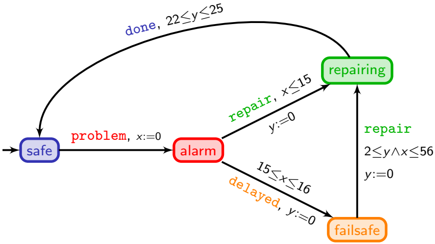
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

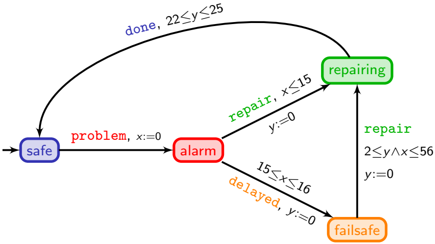
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe
⋯	15.6
	0

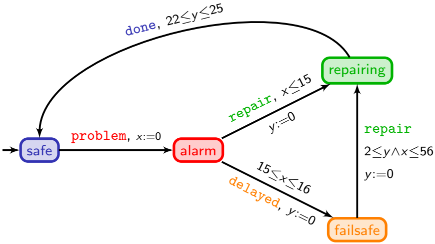
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

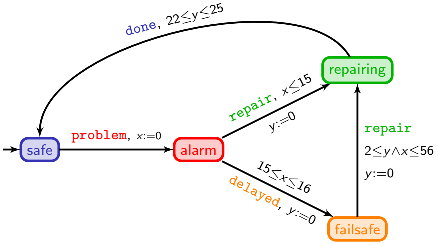
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

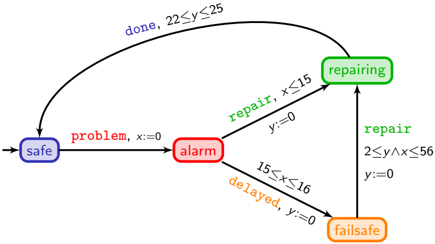
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
···	15.6		17.9		17.9		40
	0		2.3		0		22.1

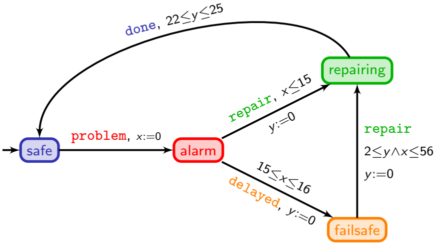
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

The model of timed automata

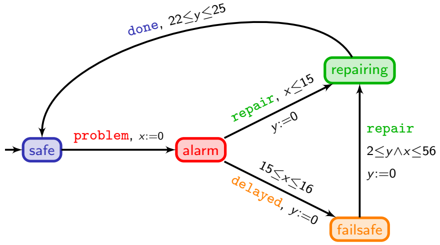


(clock) valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe		
x	0		23		0		15.6		15.6		⋯
y	0		23		23		38.6		0		

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\text{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

The model of timed automata



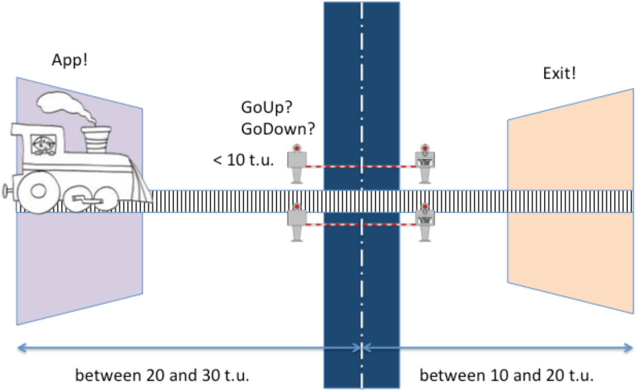
(clock) valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

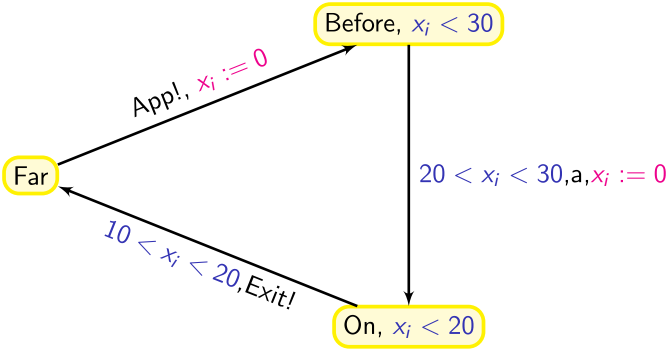
This run reads the timed word
$(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9)(\texttt{done}, 63)$

The train crossing example



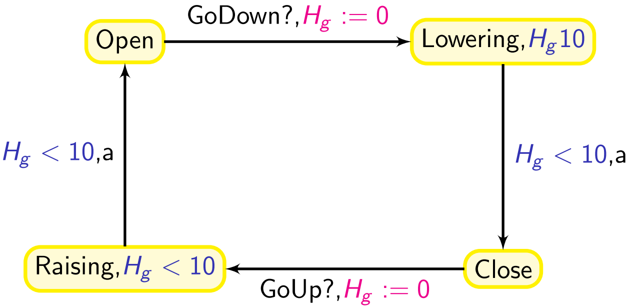
Modelling the train crossing example

Train$_i$ with $i = 1, 2, \ldots$



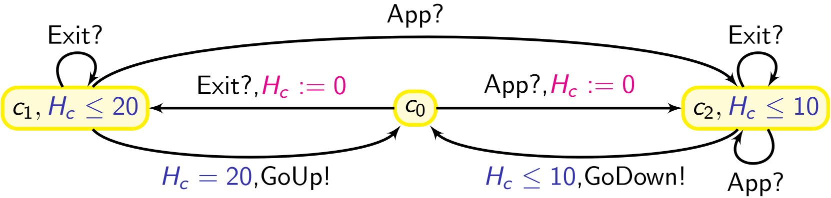
The train crossing example – cont'd

The gate:



The train crossing example – cont'd

The controller:



The train crossing example – cont'd

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?
- Is the gate always closed for less than 5 minutes?

Back to the task graph scheduling problem

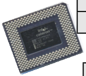
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
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idle	20 Watts
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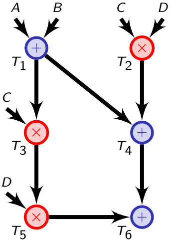
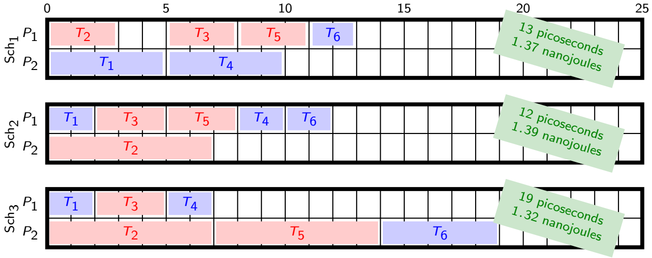
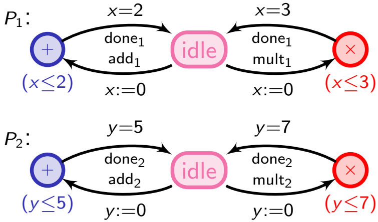




Modelling the task graph scheduling problem

Modelling the task graph scheduling problem

- Processors



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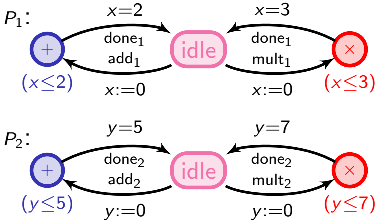
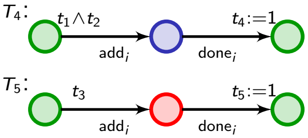


- Tasks



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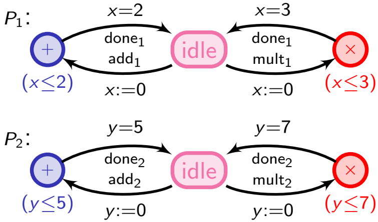

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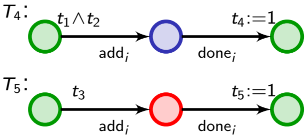

$\rightsquigarrow$ build the synchronized product of all these automata

$$(P_1 \parallel P_2) \parallel_s (T_1 \parallel T_2 \parallel \cdots \parallel T_6)$$

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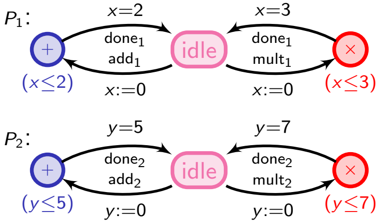

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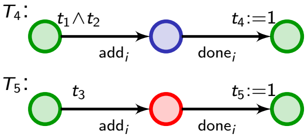

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A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

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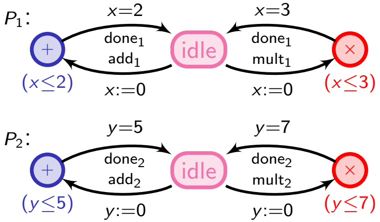

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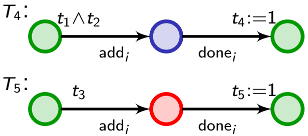

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Questions one can ask

- Can the computation be made in no more than 10 time units?
- Is there a scheduling along which no processor is ever idle?
- ...

What we have so far

- A model which can adequately represent systems with real-time constraint...
- ... on which we can ask relevant questions

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Interesting problems

- Which semantics?
  (and be aware of the limits of the choice)
- Algorithms for automatic verification

Discrete-time semantics

...because computers are digital!

[Alur91] Alur. Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

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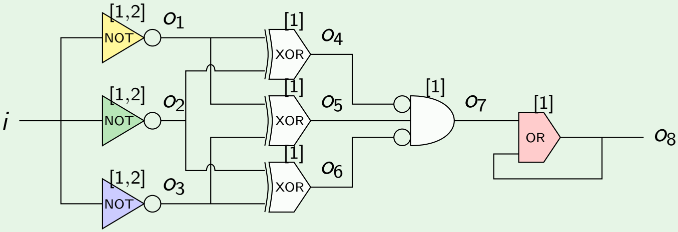
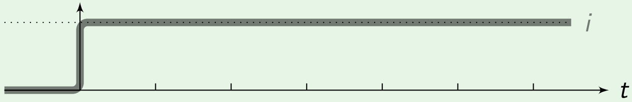
Example [Alur91]



- under discrete-time, the output is always 0:



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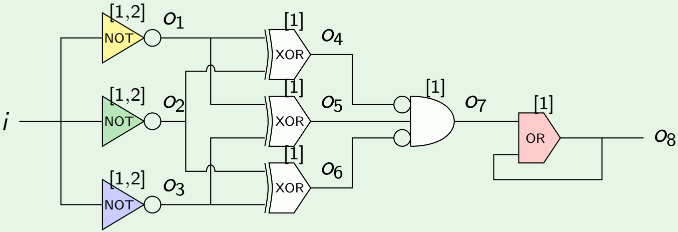

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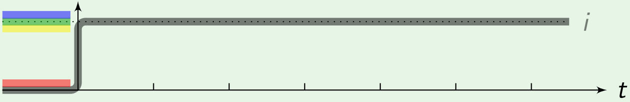

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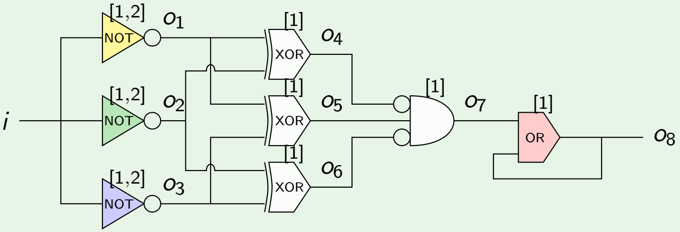

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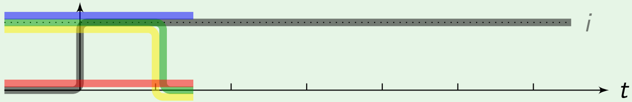

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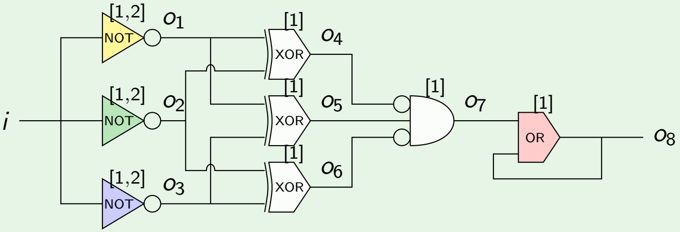

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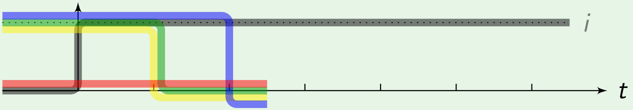

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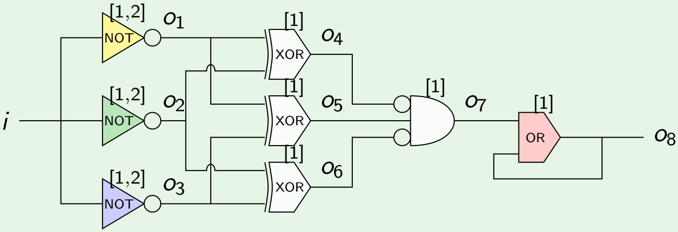

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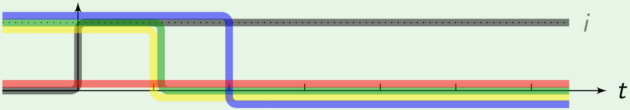

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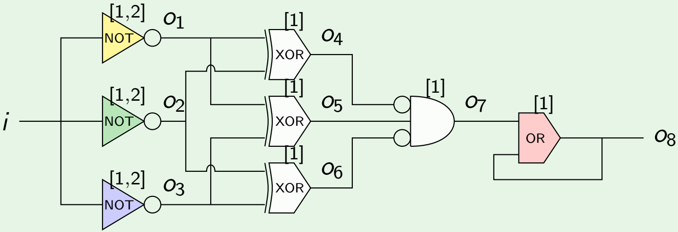
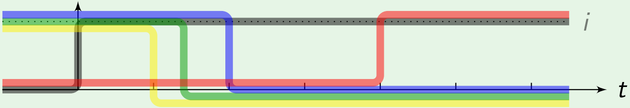


- under continuous-time, the output can be 1:



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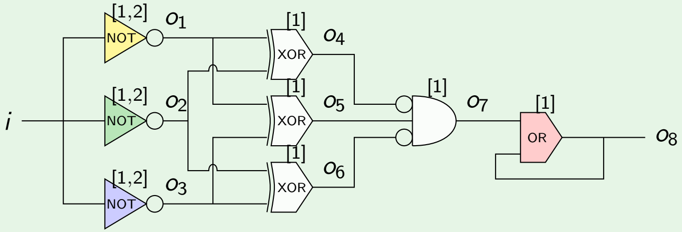

Finding the correct granularity (if one exists) is hard!

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Continuous-time semantics

...real-time models for real-time systems!

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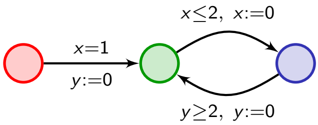

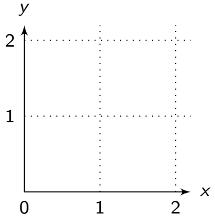

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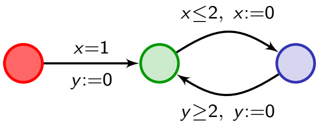

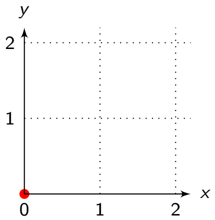

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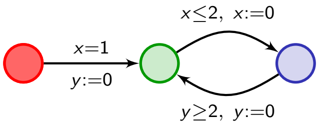

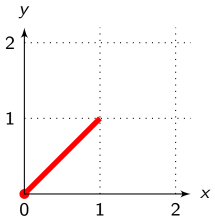

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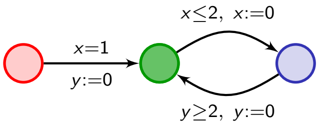

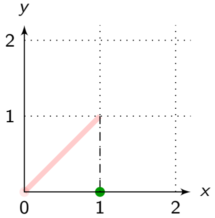

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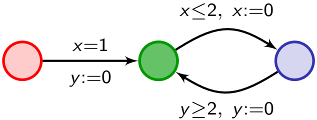

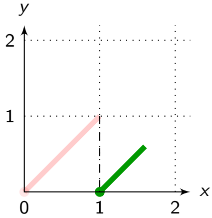

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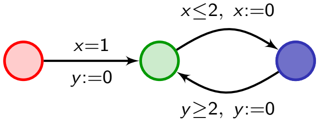

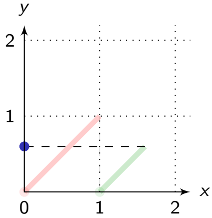

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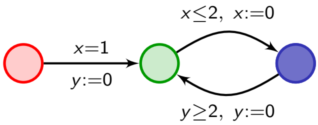

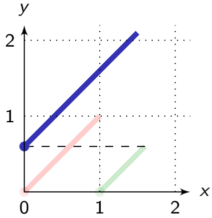

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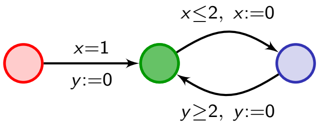

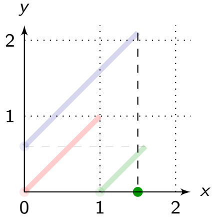

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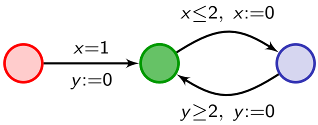

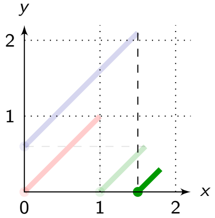

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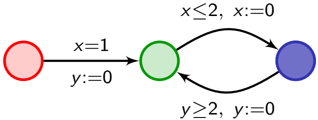

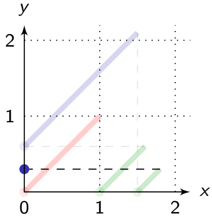

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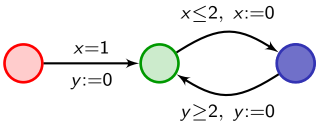

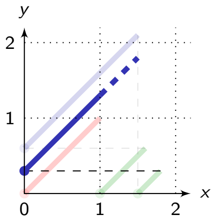

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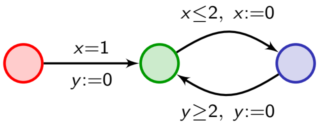

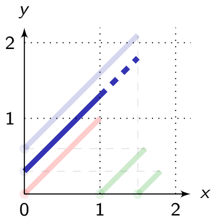

We will focus on the continuous-time semantics, since this is an adequate abstraction of real-time systems

Continuous-time semantics

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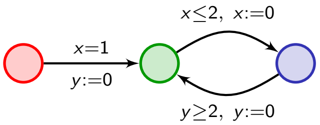

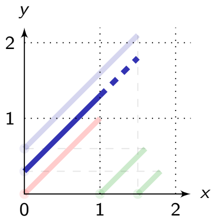

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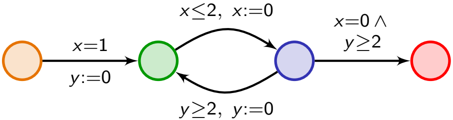
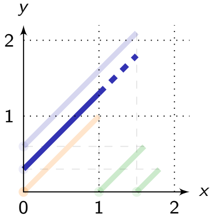
Known limits: robustness issues (we will comment on that later)

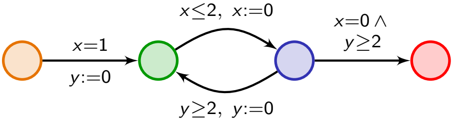
Analyzing timed automata

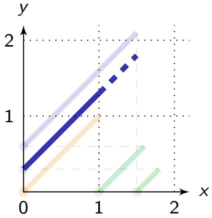




Can we reach state ?

Analyzing timed automata

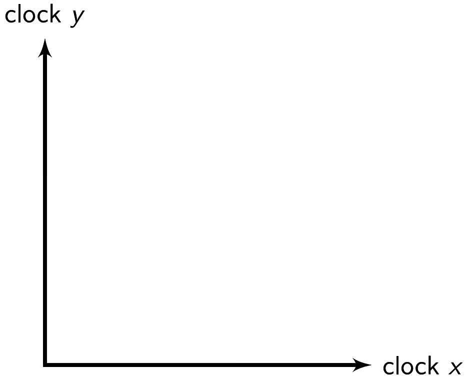




Can we reach state ?

- Problem: the set of configurations is infinite
  $\rightsquigarrow$ classical methods for finite-state systems cannot be applied
- Positive key point: variables (clocks) increase at the same speed

Crux idea: Region abstraction



[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

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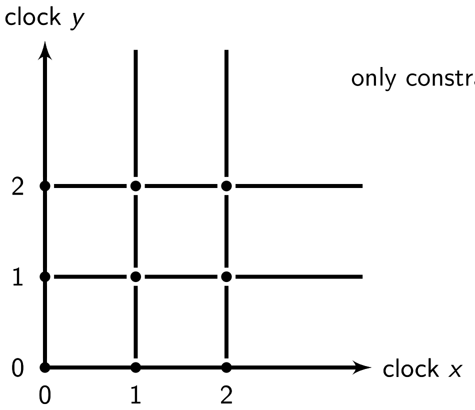

only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

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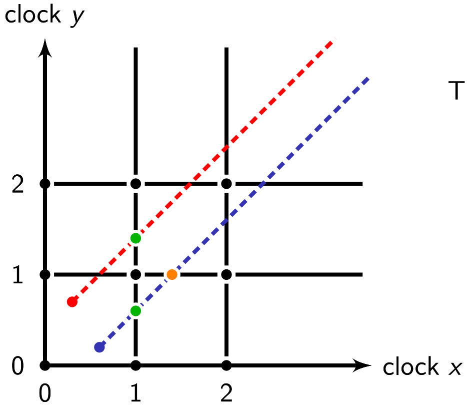

The path
- can be fired from
- cannot be fired from

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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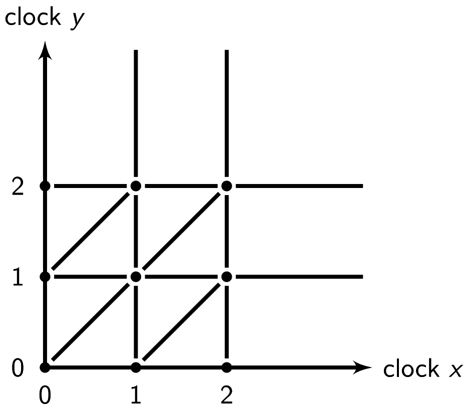

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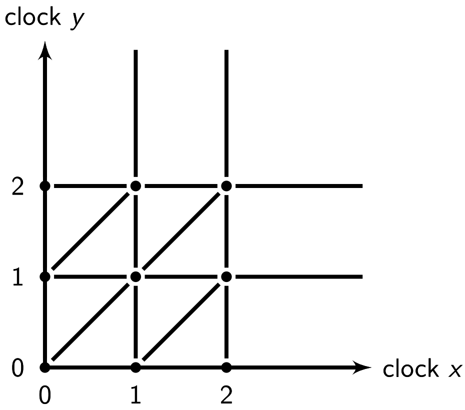

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$\rightsquigarrow$ an equivalence of finite index

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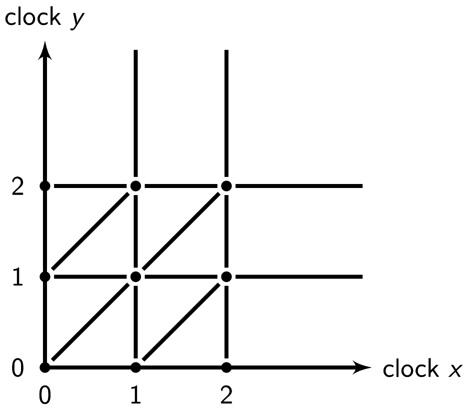

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a time-abstract bisimulation

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This is a relation between • and • such that:

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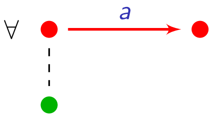

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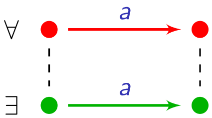

Time-abstract bisimulation

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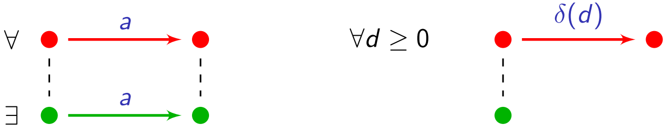

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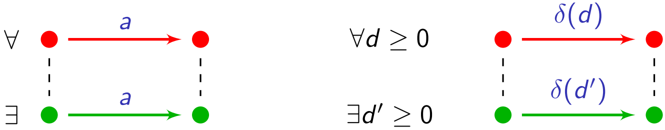

Time-abstract bisimulation

This is a relation between • and • such that:

$$\begin{tikzcd} \forall & \bullet \arrow[r, "a"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists & \bullet \arrow[r, "a"] & \bullet \end{tikzcd} \qquad \begin{tikzcd} \forall d \geq 0 & \bullet \arrow[r, "\delta(d)"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists d' \geq 0 & \bullet \arrow[r, "\delta(d')"] & \bullet \end{tikzcd}$$

... and vice-versa (swap • and •).

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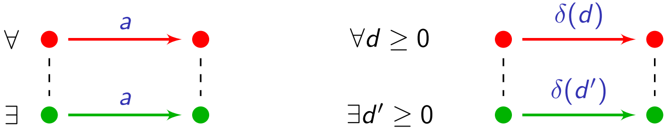

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Consequence

$$\forall \quad (\ell_1, v_1) \xrightarrow{d_1, a_1} (\ell_2, v_2) \xrightarrow{d_2, a_2} (\ell_3, v_3) \xrightarrow{d_3, a_3} \quad \cdots$$

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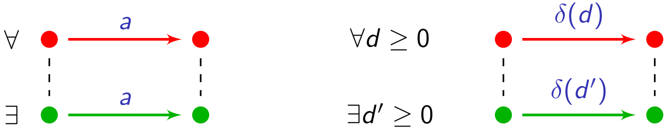

... and vice-versa (swap • and •).

Consequence

$$
\begin{tikzcd}
\forall \quad (\ell_1, v_1) \arrow[r, "d_1, a_1"] \arrow[d] & (\ell_2, v_2) \arrow[r, "d_2, a_2"] \arrow[d] & (\ell_3, v_3) \arrow[r, "d_3, a_3"] \arrow[d] & \cdots \\
(\ell_1, R_1) \arrow[r, "a_1"] & (\ell_2, R_2) \arrow[r, "a_2"] & (\ell_3, R_3) \arrow[r, "a_3"] & \cdots
\end{tikzcd}
$$

with $v_i \in R_i$

Time-abstract bisimulation

This is a relation between $\bullet$ and $\bullet$ such that:

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$\forall v_1' \in R_1$

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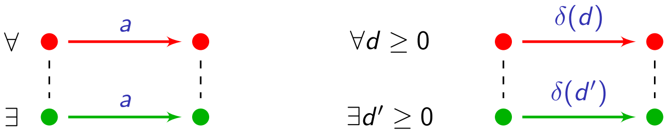

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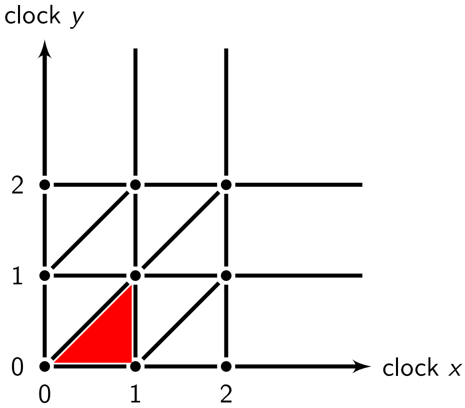
\forall v'_1 \in R_1 \; \exists & (\ell_1, v'_1) \arrow[r, "d'_1, a_1"] & (\ell_2, v'_2) \arrow[r, "d'_2, a_2"] & (\ell_3, v'_3) \arrow[r, "d'_3, a_3"] & \cdots & \text{with } v'_i \in R_i
\end{tikzcd}
$$

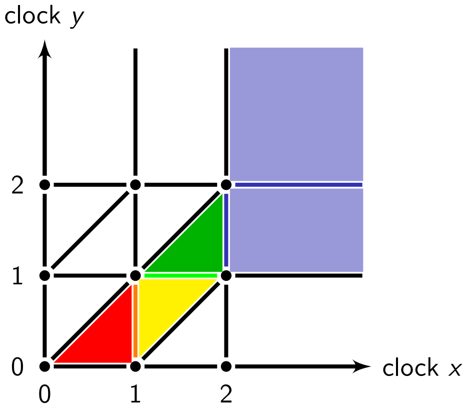
The region abstraction



- region R defined by:

$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

The region abstraction



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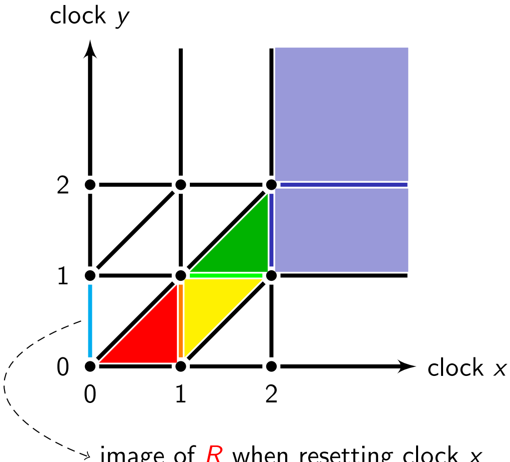


- region R defined by:

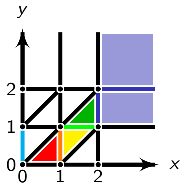
$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

- time successors of R

image of R when resetting clock x

The construction of the region graph

It "mimicks" the behaviours of the clocks.



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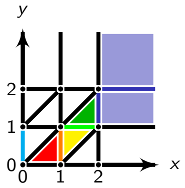
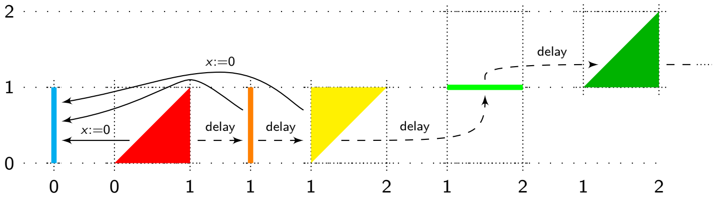
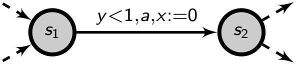


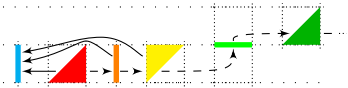


Region automaton $\equiv$ finite bisimulation quotient



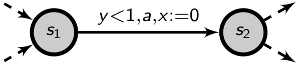
timed automaton

$\otimes$



region graph

Region automaton ≡ finite bisimulation quotient



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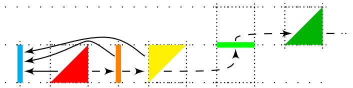

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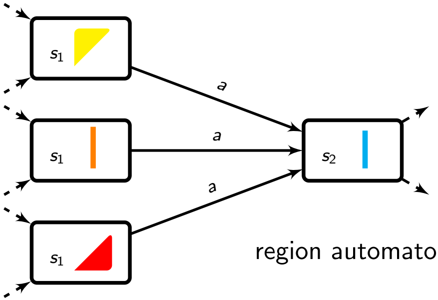

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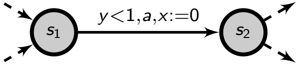

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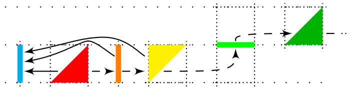

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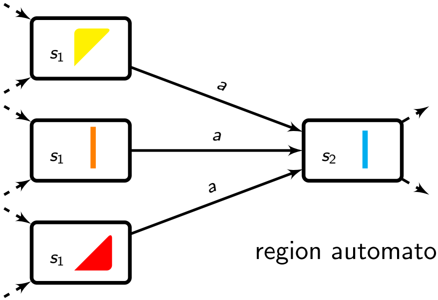
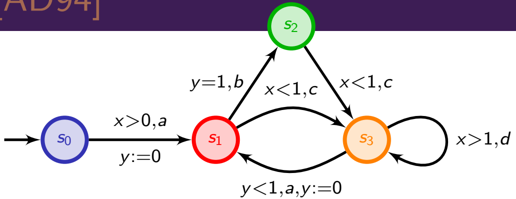


region automaton

language(reg. aut.) = UNTIME(language(timed aut.))

An example [AD94]



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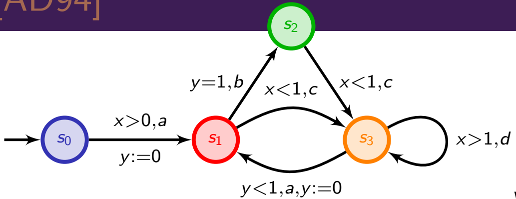

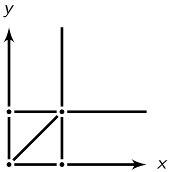

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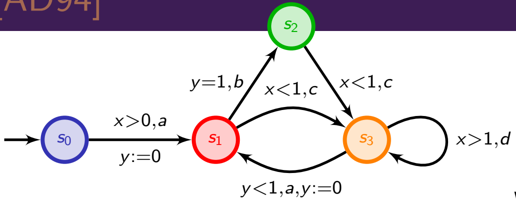
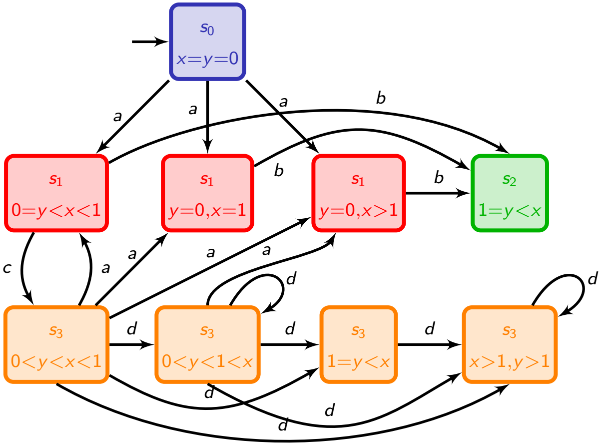
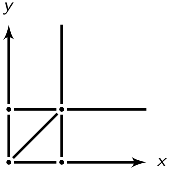
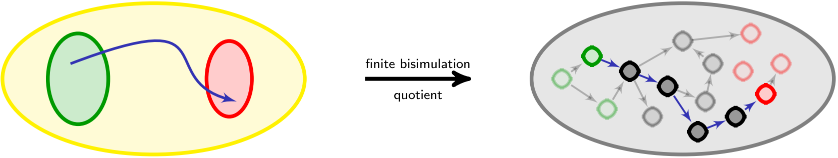


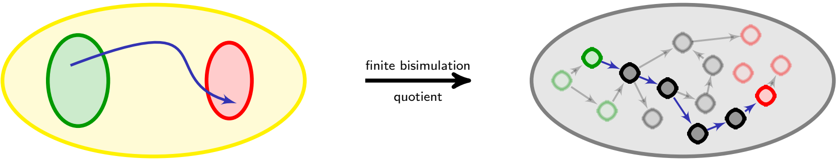






timed automaton

large (but finite) automaton
(region automaton)



timed automaton

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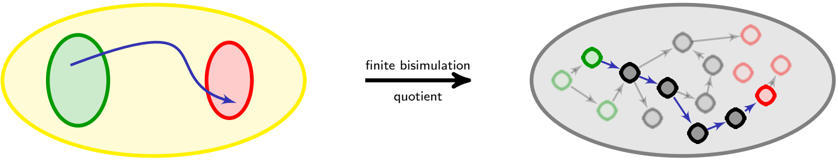
(region automaton)

- large: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is:

$$\prod_{x \in X} (2M_x + 2) \cdot |X|! \cdot 2^{|X|}$$



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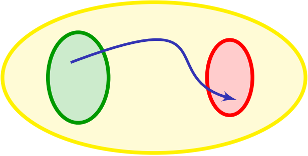
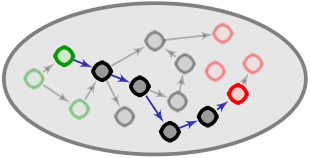


timed automaton

finite bisimulation

quotient



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- It can be used to check for:
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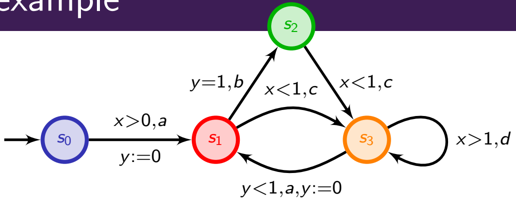
- Problems with Zeno behaviours?
  (infinitely many actions in bounded time)

Back to the example



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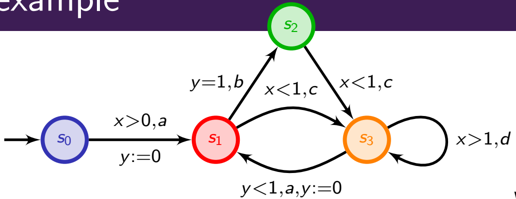

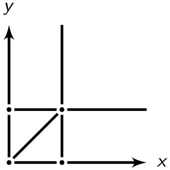

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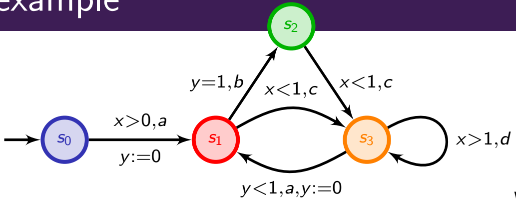

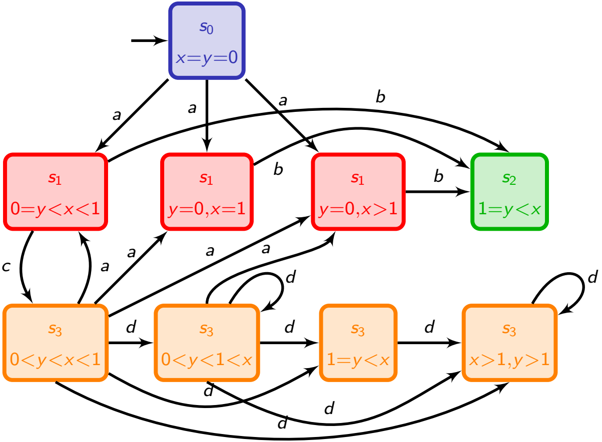

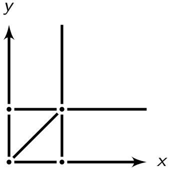

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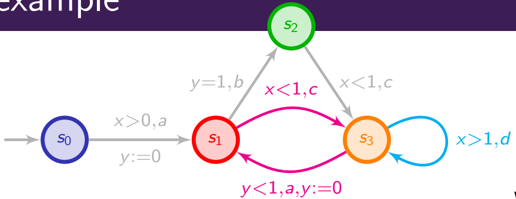
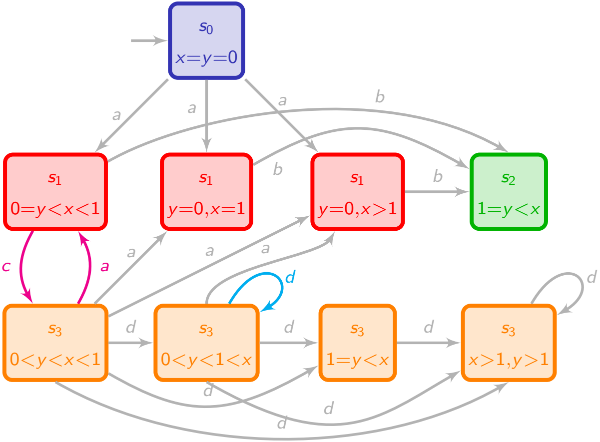
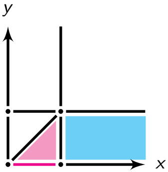
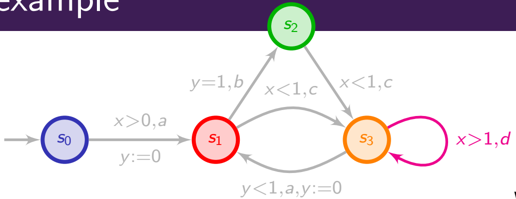


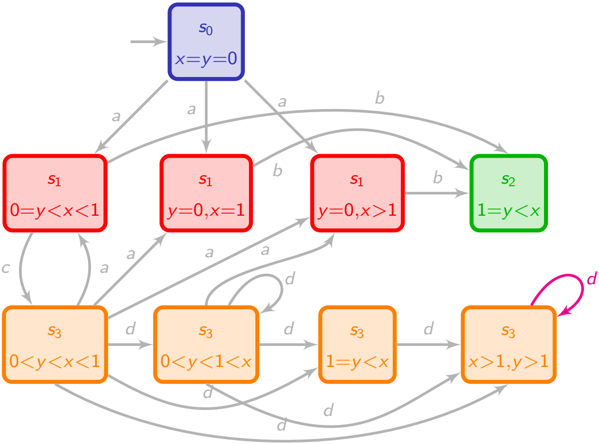


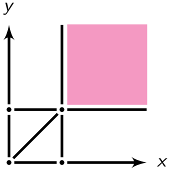


Zeno cycles

Back to the example







Cycles with non-Zeno behaviours

Complexity issues

Theorem [AD90,AD94]

The emptiness problem for timed automata is decidable and PSPACE-complete. It even holds for two-clock timed automata [FJ13]. It is NLOGSPACE-complete for one-clock timed automata [LMS04].

Skip

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.
[LMS04] Laroussinie, Markey, Schnoebelen. Model checking timed automata with one or two clocks *(CONCUR'04)*.
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete *(ICALP'13)*.

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- PSPACE upper bound: guess a path in the region automaton

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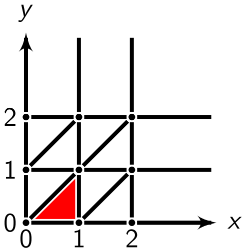

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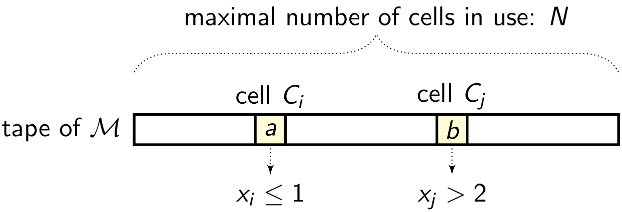
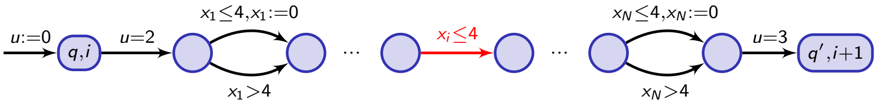


Example of the simulation of a rule $(q, a, b, q', \rightarrow)$:



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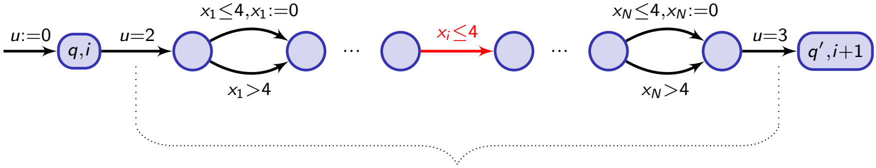

constraint $x_j \leq 4$: cell j contains an a

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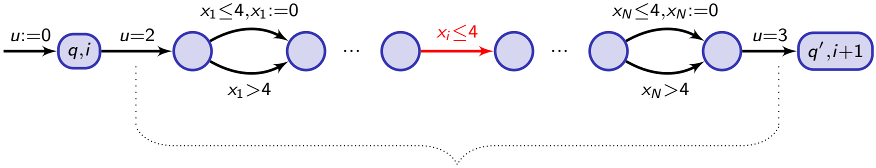

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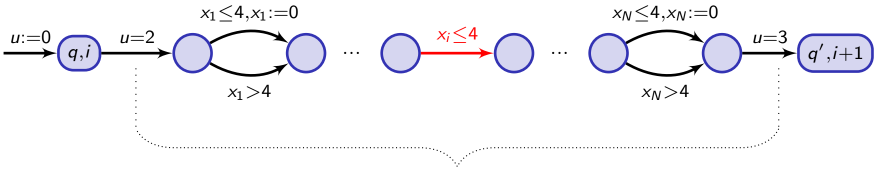

constraint $x_j{\leq}4$: cell j contains an a
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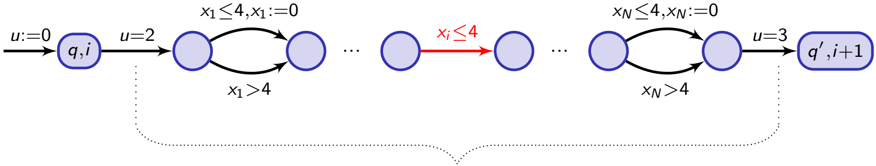
reset of clock x_j: the new content is an a

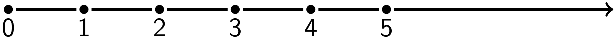
Example of the simulation of a rule $(q, a, b, q', \rightarrow)$:



constraint $x_j \leq 4$: cell j contains an a
constraint $x_j > 4$: cell j contains a b

reset of clock x_j: the new content is an a
no reset of clock x_j: the new content is a b

The case of single-clock timed automata



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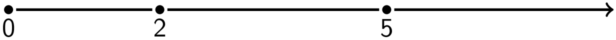

if only constants 0, 2 and 5 are used

Discussion

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- Note however that it might be hard to prove there is a finite bisimulation quotient!

What about the practice?

- the region automaton is never computed
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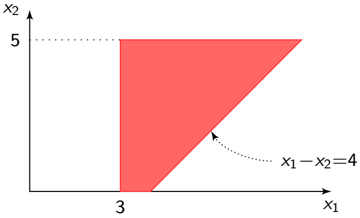
- Symbolic representation: zones

$$Z \;=\; (x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$



$$\begin{array}{cc} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

DBM: Difference Bound Matrice [BM83,Dill89]

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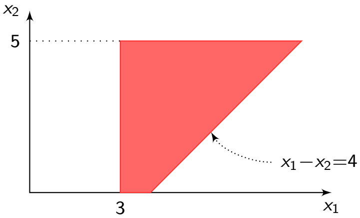

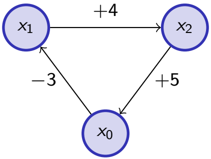

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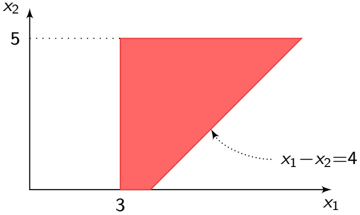

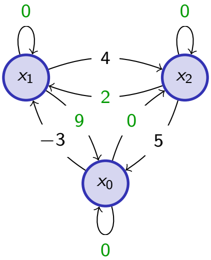

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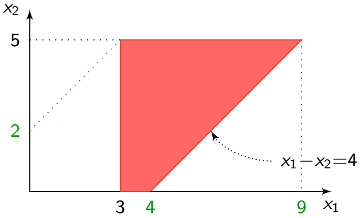

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"normal form"

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- resets of clocks to 0 only; we can reset to integral values as well
  - more involved updates can be used as well, but they don't interact very well with diagonal constraints. So one needs to be careful

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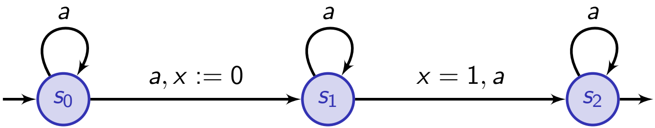
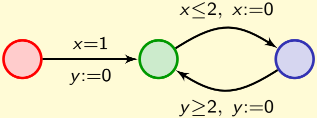
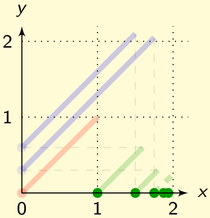
- One cannot complement nor determinize timed automata



An important issue: Robustness and implementability





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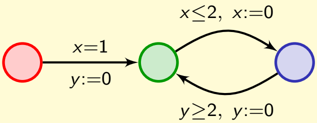

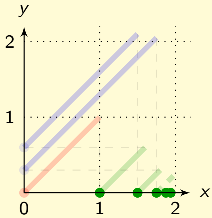

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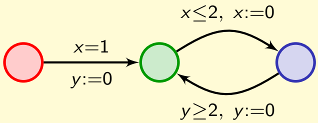

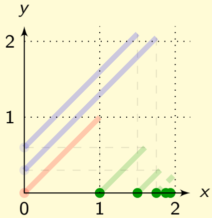

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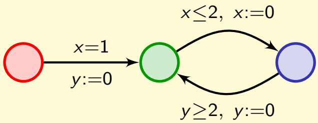

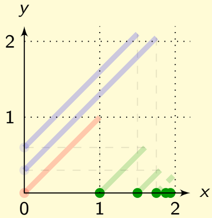

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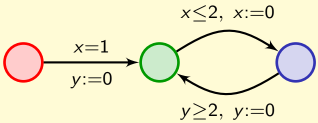

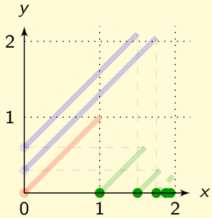

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A survey: [BMS13]

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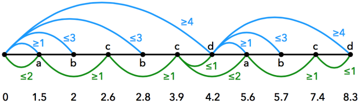
Theoretical recent developments

- Tree automata technics for timed automata analysis [AGK16,AGKS17]



  - Write behaviours as graphs with timing constraints
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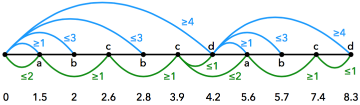

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- Compute and use the reachability relation [CJ99,QSW17]

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[CJ99] Comon, Jurski. Timed Automata and the Theory of Real Numbers *(CONCUR'99)*.
[QSW17] Quaas, Shirmohammadi, Worrell. Revisiting Reachability in Timed Automata *(LICS'17)*.

Outline

Back to the task-graph scheduling problem

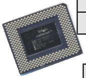
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

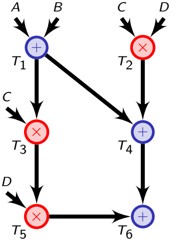
energy	
idle	10 Watt
in use	90 Watts

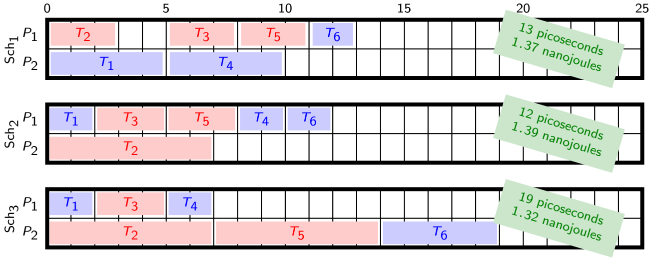
P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts





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×	7 picoseconds

energy

How to model energy consumption?

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- A possible solution: use hybrid automata

  a discrete control (the mode of the system)

  $+$ continuous evolution of the variables within a mode

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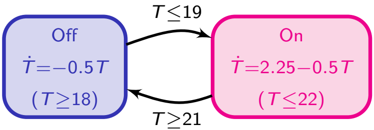

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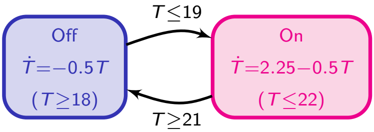

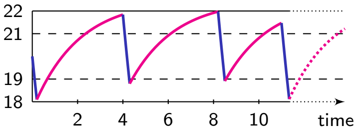

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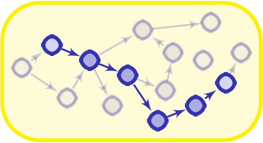

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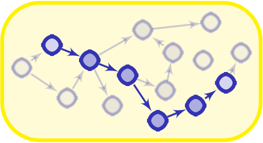

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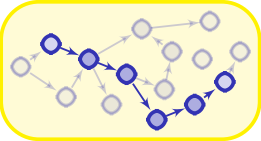

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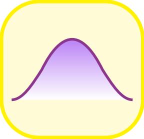

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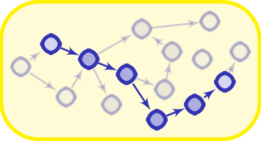

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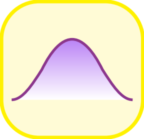
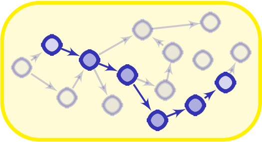


Easy...

Ok... but?



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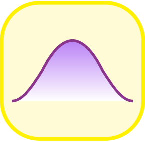
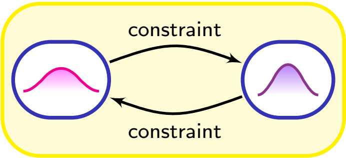
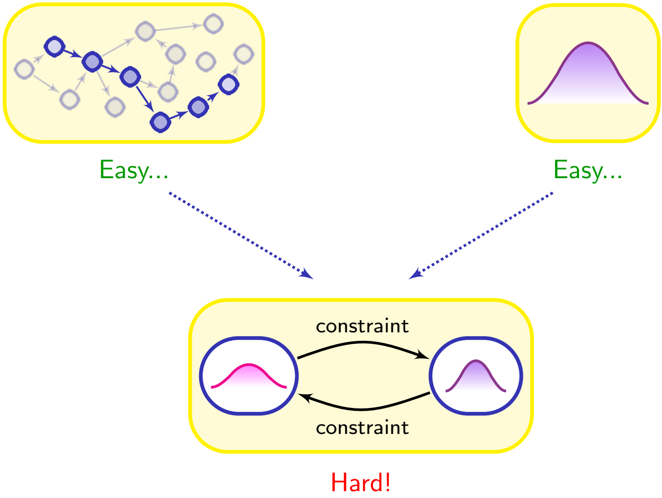


Easy...



Ok... but?



Modelling resources in timed systems

- System resources might be relevant and even crucial information
  - energy consumption,
  - memory usage,
  - ...
  - price to pay,
  - bandwidth,

  $\rightsquigarrow$ timed automata are not powerful enough!

- A possible solution: use hybrid automata

Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

Modelling resources in timed systems

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Theorem [HKPV95]

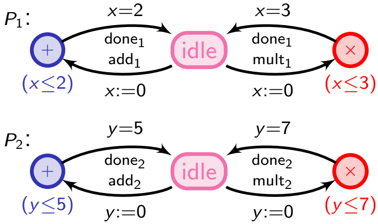
The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system
  hybrid variables are observer variables

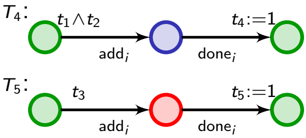
[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Modelling the task graph scheduling problem

- Processors



- Tasks



Modelling the task graph scheduling problem

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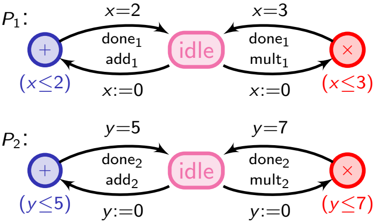
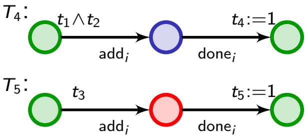
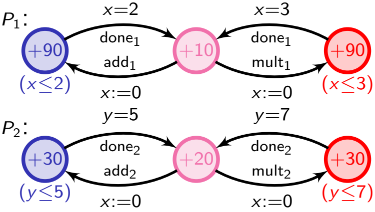


- Tasks

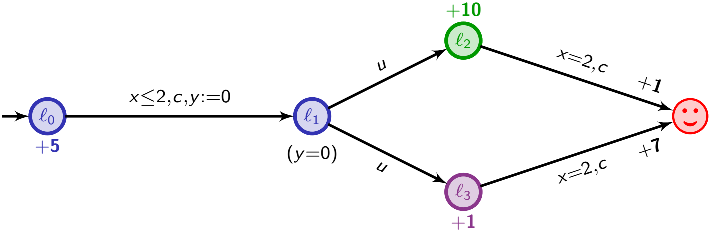


- Modelling energy



A good schedule is a path in the product automaton with a low cost

Weighted/priced timed automata [ALP01,BFH+01]



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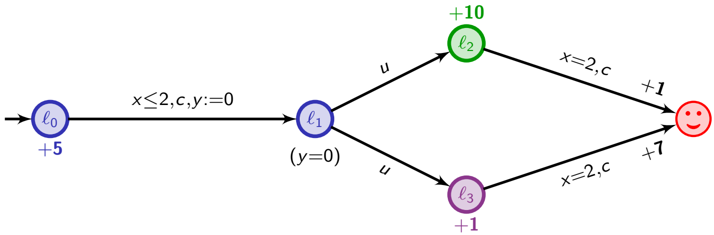
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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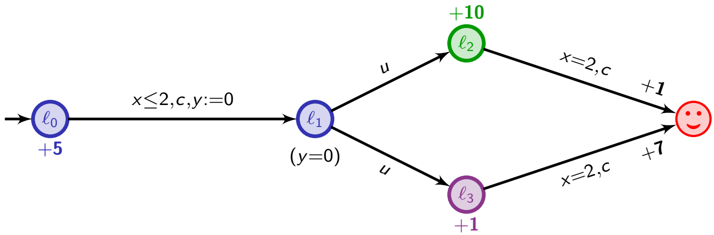

$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost :

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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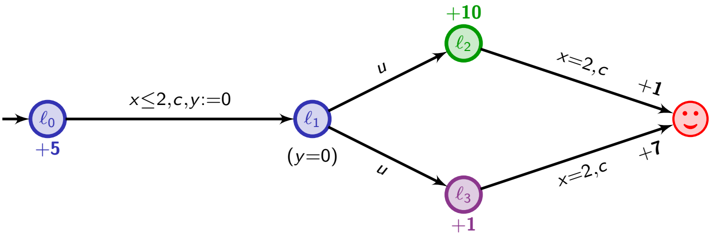
Weighted/priced timed automata [ALP01,BFH+01]



$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost : 6.5

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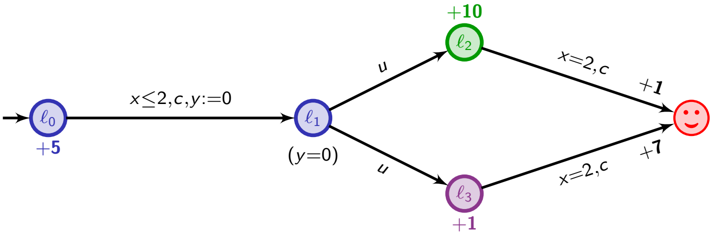
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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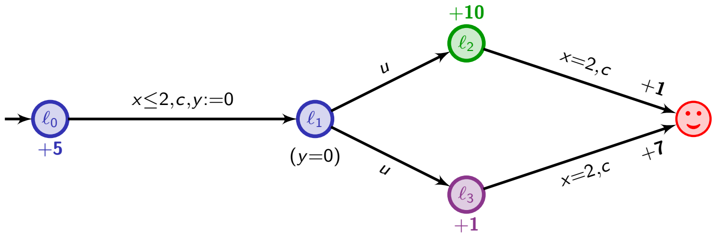
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : $6.5 + 0 + 0$

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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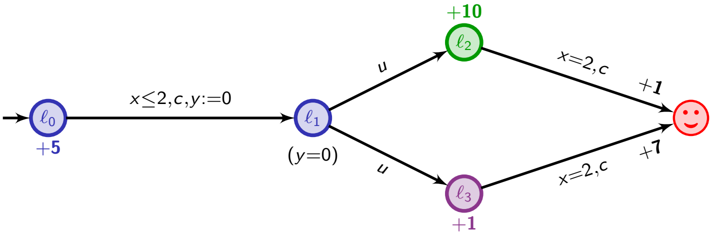

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7			

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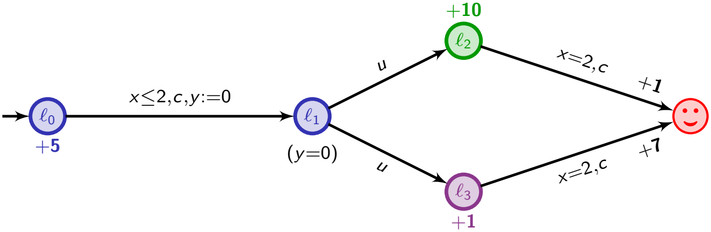
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7	+	7	

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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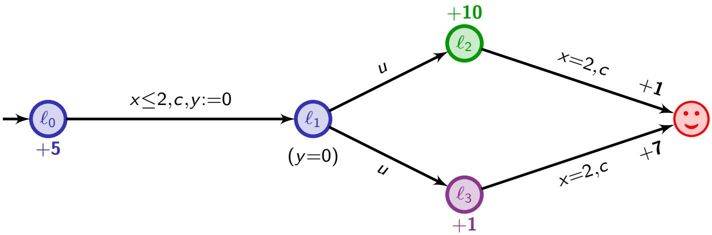
Weighted/priced timed automata [ALP01,BFH+01]

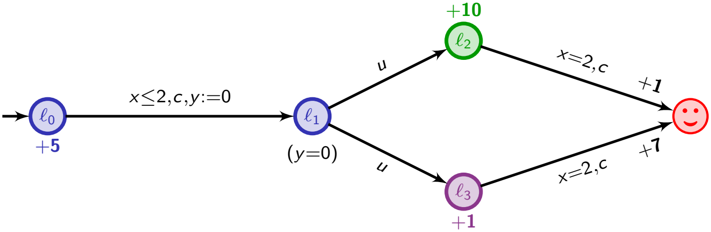


	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7 = 14.2

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

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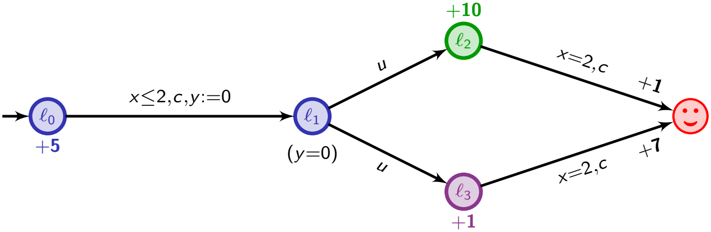

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$$5t + 10(2 - t) + 1$$

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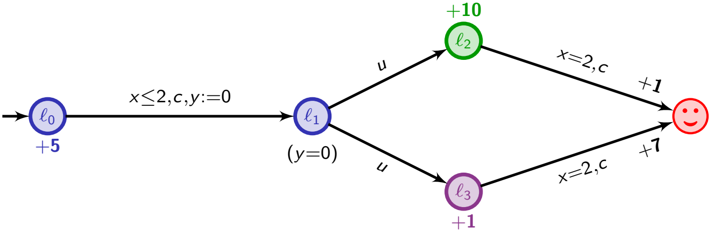
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

$$5t + 10(2 - t) + 1\ ,\ 5t + (2 - t) + 7$$

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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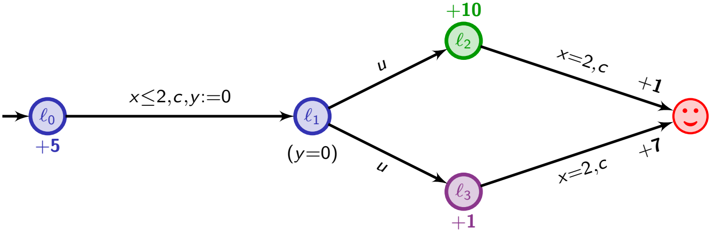
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

$$\min\left(\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ \right)$$

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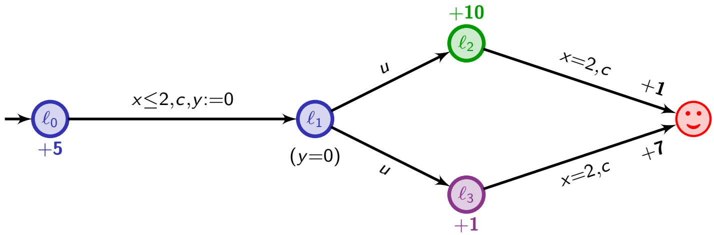
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺?

$$\inf_{0\leq t\leq 2} \min\left(5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7 \right) = 9$$

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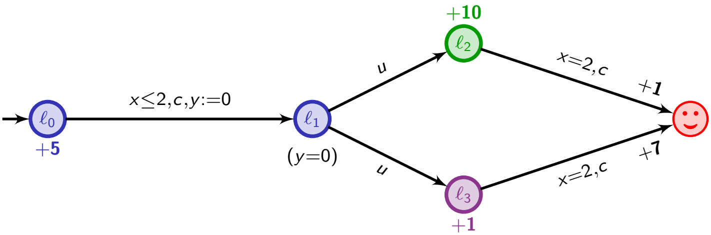
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

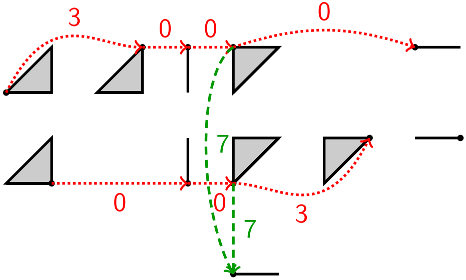
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

In weighted timed automata, the optimal cost is an integer and can be computed in PSPACE.

- Technical tool: a refinement of the regions, the corner-point abstraction



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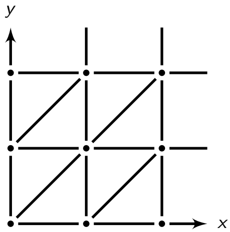
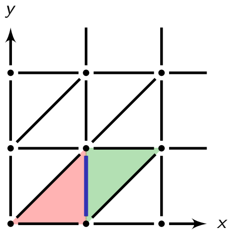
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

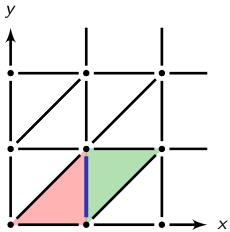
Technical tool: the corner-point abstraction



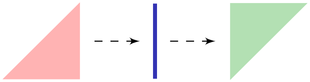
Technical tool: the corner-point abstraction



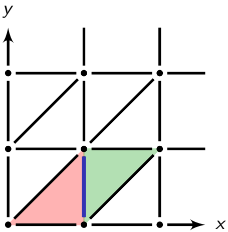
Technical tool: the corner-point abstraction



Abstract time successors:



Technical tool: the corner-point abstraction



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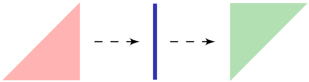
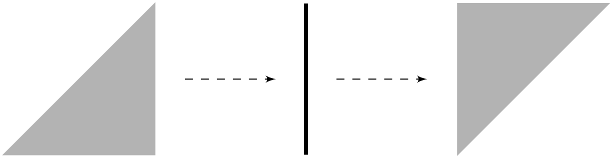


Concrete time successors:



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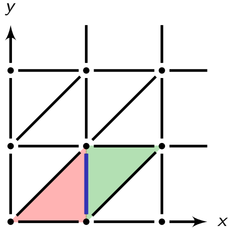
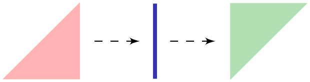
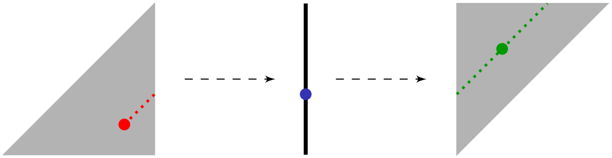


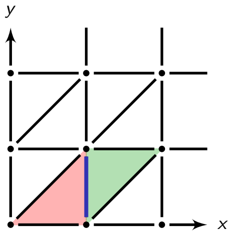
Abstract time successors:



Concrete time successors:



Technical tool: the corner-point abstraction



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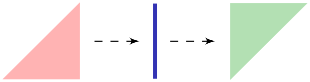

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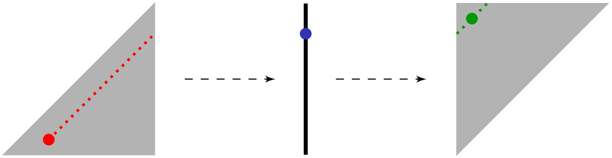
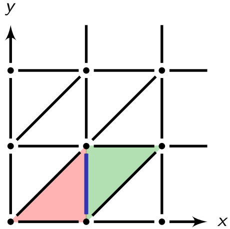


Technical tool: the corner-point abstraction



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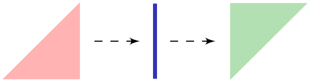

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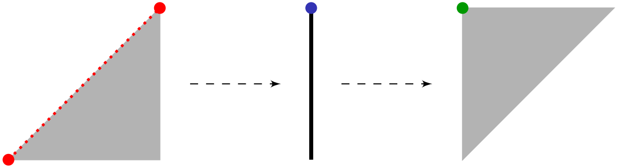
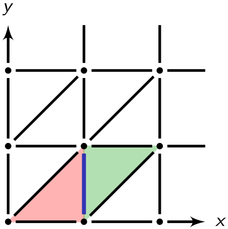


Technical tool: the corner-point abstraction



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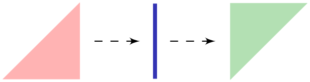
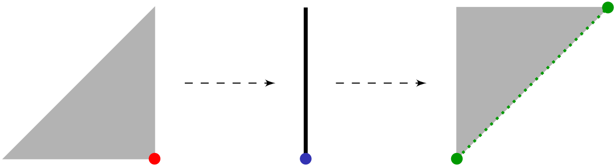
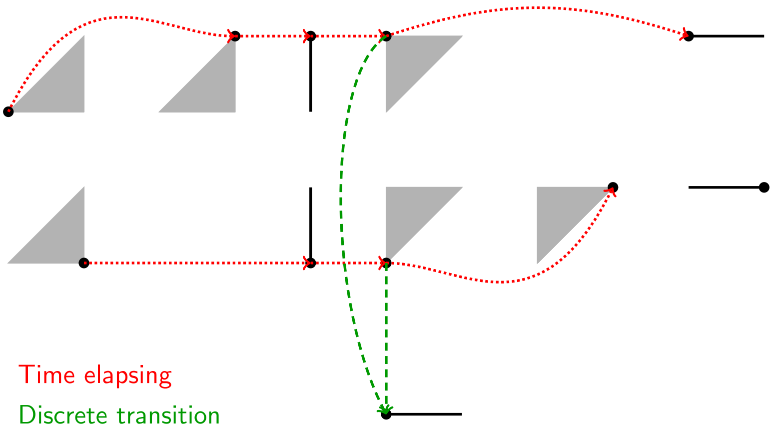


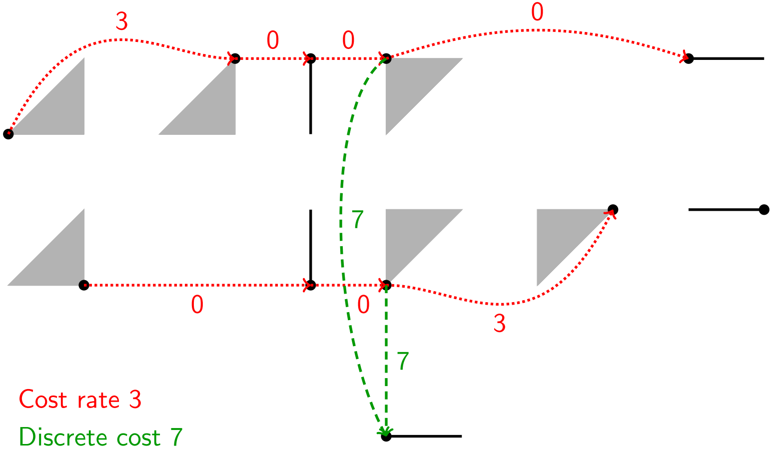
Concrete time successors:



Technical tool: the corner-point abstraction



Technical tool: the corner-point abstraction



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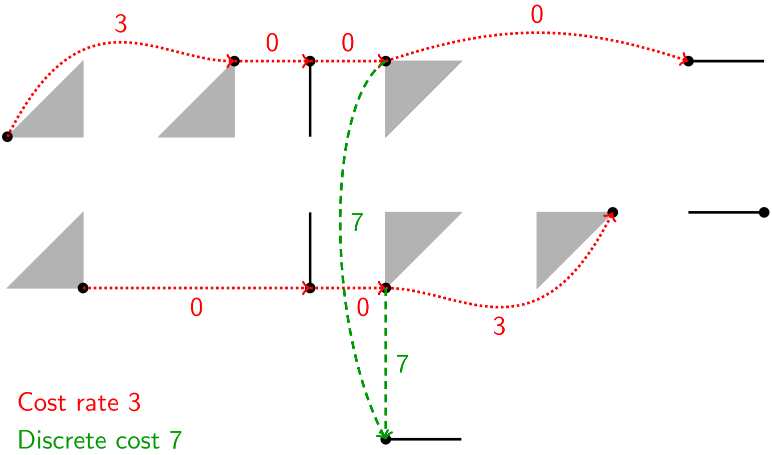

Optimal cost in the weighted graph
= optimal cost in the weighted timed automaton!

From timed to discrete behaviours

Optimal reachability as a linear programming problem

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow[x \leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \left\{ \begin{array}{l} t_1 + t_2 \leq 2 \\ \\ \end{array} \right.$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 \\ t_2+t_3+t_4\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{ccccccccccc} & T_1 & & T_2 & & T_3 & & T_4 & & T_5 & \\ \circ & \xrightarrow[y:=0]{t_1} & \circ & \xrightarrow[x\leq 2]{t_2} & \circ & \xrightarrow{t_3} & \circ & \xrightarrow[y\geq 5]{t_4} & \circ & \xrightarrow{t_5} & \circ \quad \cdots \end{array} \quad \left\{ \begin{array}{ll} t_1+t_2\leq 2 & T_2\leq 2 \\ t_2+t_3+t_4\geq 5 & T_4-T_1\geq 5 \end{array} \right.$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{cccccc} & T_1 & T_2 & T_3 & T_4 & T_5 \end{array}$$

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 & T_2\leq 2 \\ t_2+t_3+t_4\geq 5 & T_4-T_1\geq 5 \end{cases}$$

Lemma

Let Z be a bounded zone and f be a function

$$f : (T_1, ..., T_n) \mapsto \sum_{i=1}^{n} c_i T_i + c$$

well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{cccccc} T_1 & T_2 & T_3 & T_4 & T_5 & \\ \end{array}$$

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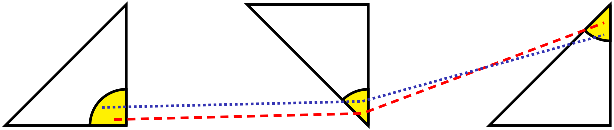
$\rightsquigarrow$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\text{cp}}$ such that

$$\text{cost}(\Pi) \leq \text{cost}(\pi)$$

[Π is a "corner-point projection" of π]

From discrete to timed behaviours

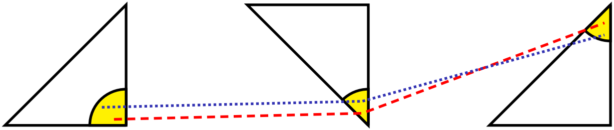
Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$,

From discrete to timed behaviours

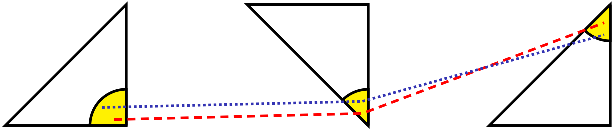
Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

From discrete to timed behaviours

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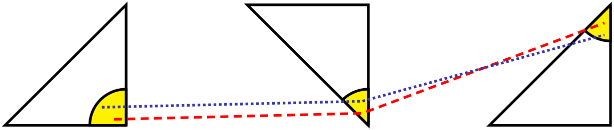

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For every $\eta > 0$, there exists $\varepsilon > 0$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\mathsf{cost}(\Pi) - \mathsf{cost}(\pi_\varepsilon)| < \eta$$

Use of the corner-point abstraction

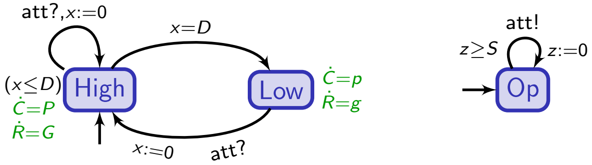
It is a very interesting abstraction, that can be used in several other contexts:

- for mean-cost optimization [BBL04,BBL08]
- for discounted-cost optimization [FL08]
- for all concavely-priced timed automata [JT08]
- for deciding frequency objectives [BBBS11,Sta12]
- ...

Skip

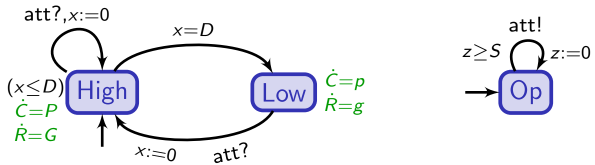
[BBL04] Bouyer, Brinksma, Larsen. Staying Alive As Cheaply As Possible *(HSCC'04)*.
[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.
[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.
[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.
[BBBS11] Bertrand, Bouyer, Brihaye, Stainer. Emptiness and universality problems in timed automata with positive frequency *(ICALP'11)*.
[Sta12] Stainer. Frequencies in forgetful timed automata *(FORMATS'12)*.

Going further 1: mean-cost optimization



[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

Going further 1: mean-cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{mean-cost}(\pi) = \limsup_{n \to +\infty} \frac{\text{cost}(\pi_n)}{\text{reward}(\pi_n)}$$

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

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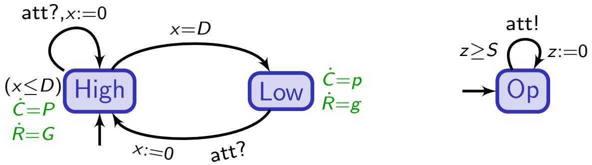

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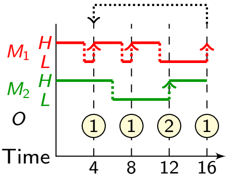
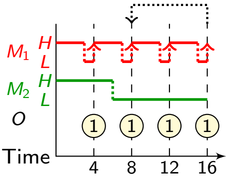


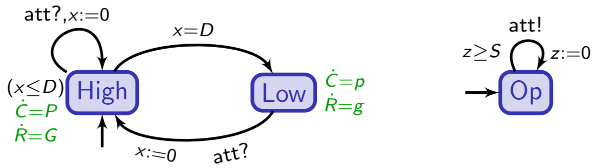
Schedule with ratio ≈1.455



Schedule with ratio ≈1.478

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

Going further 1: mean-cost optimization



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$$\text{mean-cost}(\pi) = \limsup_{n \to +\infty} \frac{\text{cost}(\pi_n)}{\text{reward}(\pi_n)}$$

Theorem [BBL08]

In weighted timed automata, the optimal mean-cost can be compute in PSPACE.

$\rightsquigarrow$ the corner-point abstraction can be used

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

From timed to discrete behaviours

- **Finite behaviours:** based on the following property

Lemma

Let Z be a bounded zone and f be a function

$$f : (t_1, \ldots, t_n) \mapsto \frac{\sum_{i=1}^{n} c_i t_i + c}{\sum_{i=1}^{n} r_i t_i + r}$$

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$\rightsquigarrow$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathrm{cp}}$ s.t. mean-cost$(\Pi) \leq$ mean-cost(π)

From timed to discrete behaviours

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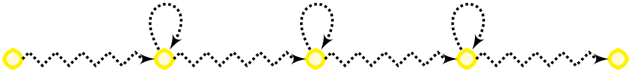
$$\text{mean-cost}(\Pi) \leq \text{mean-cost}(\pi)$$

- **Infinite behaviours:** decompose each sufficiently long projection into cycles:



The (acyclic) linear part will be negligible!

From timed to discrete behaviours

- **Finite behaviours:** based on the following property

Lemma

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well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

$\leadsto$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathrm{cp}}$ s.t.

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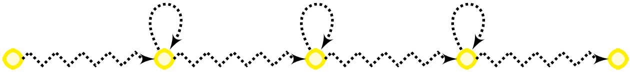
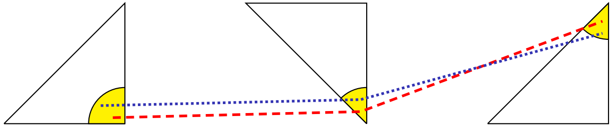


The (acyclic) linear part will be negligible!

$\leadsto$ the optimal cycle of $\mathcal{A}_{\mathrm{cp}}$ is better than any infinite path of $\mathcal{A}$!

From discrete to timed behaviours

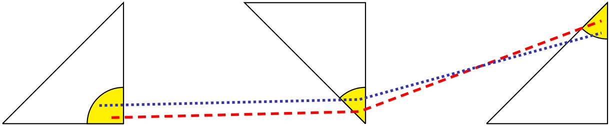
Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$,

From discrete to timed behaviours

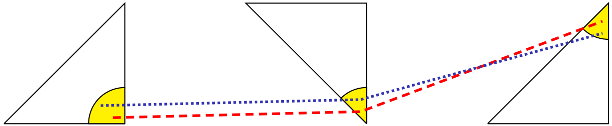
Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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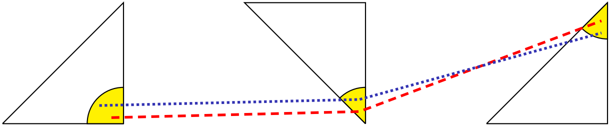

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Going further 2: concavely-priced cost functions

$\rightsquigarrow$ A general abstract framework for quantitative timed systems

Theorem [JT08]

In concavely-priced timed automata, optimal cost is computable, if we restrict to quasi-concave cost functions. For the following cost functions, the (decision) problem is even PSPACE-complete:

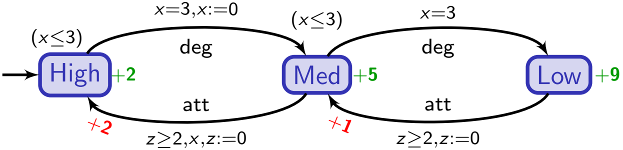
- optimal-time and optimal-cost reachability;
- optimal discrete discounted cost;
- optimal mean-cost.

$\rightsquigarrow$ the corner-point abstraction can be used

[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.

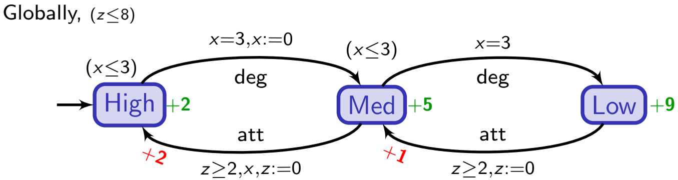
Going further 3: discounted-time cost optimization

Globally, $(z \leq 8)$



[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

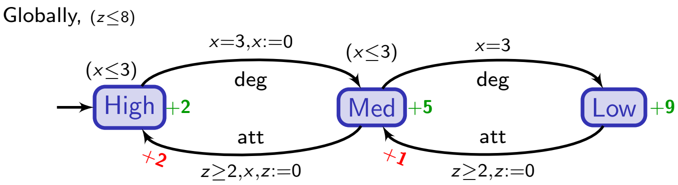
Going further 3: discounted-time cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize discounted cost over time

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



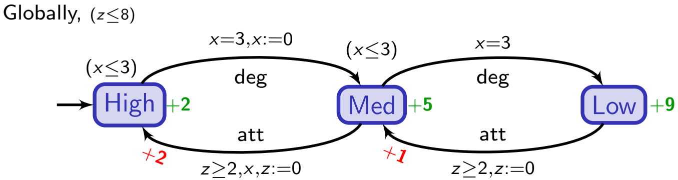
$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{discounted-cost}_{\lambda}(\pi) = \sum_{n\geq 0} \lambda^{T_n} \int_{t=0}^{\tau_{n+1}} \lambda^t \text{cost}(\ell_n)\,\mathrm{d}t + \lambda^{T_{n+1}} \text{cost}(\ell_n \xrightarrow{a_{n+1}} \ell_{n+1})$$

if $\pi = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \xrightarrow{\tau_2, a_2} \cdots$ and $T_n = \sum_{i\leq n} \tau_i$

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



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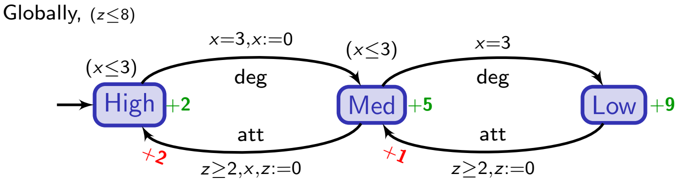

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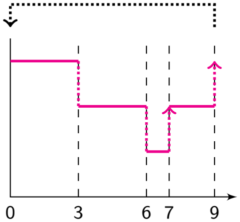
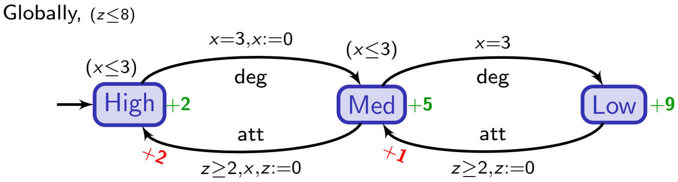


if $\lambda = e^{-1}$, the discounted cost of that infinite schedule is ≈ 2.16

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize
discounted cost over time

Theorem [FL08]

In weighted timed automata, the optimal discounted cost is computable in EXPTIME.

$\rightsquigarrow$ the corner-point abstraction can be used

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

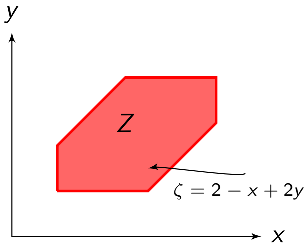
And symbolically?

- Non-obvious in general...

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.
[BCM16] Bouyer, Colange, Markey. Symbolic optimal reachability in weighted timed automata *(CAV'16)*.

And symbolically?

- Non-obvious in general...
- Only for optimal reachability



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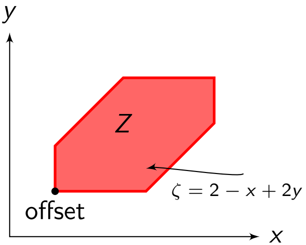
And symbolically?

- Non-obvious in general...
- Only for optimal reachability

Priced zones

priced zone = zone + affine cost function

☺ efficient representation: DBM + offset cost + affine coefficient for each clock



Represented by: zone Z
offset cost: $+4$
rate for x: -1
rate for y: $+2$

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.
[BCM16] Bouyer, Colange, Markey. Symbolic optimal reachability in weighted timed automata *(CAV'16)*.

Results

Theorem [LBB+01,RLS06]

The forward algorithm with standard inclusion is correct and terminates for **bounded** timed automata with non-negative costs.

Termination: well-quasi-order on priced zones

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.

[RLS06] Rasmussen, Larsen, Subramani. On using priced timed automata to achieve optimal scheduling *(Formal Methods in System Design)*.

Results

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- Development of an (abstract) inclusion test $\sqsubseteq_M$ on priced zones

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[BCM16] Bouyer, Colange, Markey. Symbolic Optimal Reachability in Weighted Timed Automata *(CAV'16)*.

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Termination: well-quasi-order on priced zones

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- $\mathcal{Z} \sqsubseteq_M \mathcal{Z}'$ reduces to several bilevel linear optimization problems

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.
[RLS06] Rasmussen, Larsen, Subramani. On using priced timed automata to achieve optimal scheduling *(Formal Methods in System Design)*.
[BCM16] Bouyer, Colange, Markey. Symbolic Optimal Reachability in Weighted Timed Automata *(CAV'16)*.

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Termination: well-quasi-order on priced zones

- Development of an (abstract) inclusion test $\sqsubseteq_M$ on priced zones
- $\mathcal{Z} \sqsubseteq_M \mathcal{Z}'$ reduces to several bilevel linear optimization problems

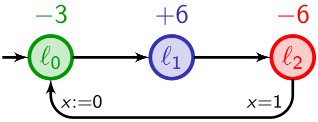
Theorem [BCM16]

The forward algorithm with inclusion test $\sqsubseteq_M$ is correct and terminates for timed automata with some conditions on the cost.
It is always better than standard inclusion for bounded timed automata.

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.
[RLS06] Rasmussen, Larsen, Subramani. On using priced timed automata to achieve optimal scheduling *(Formal Methods in System Design)*.
[BCM16] Bouyer, Colange, Markey. Symbolic Optimal Reachability in Weighted Timed Automata *(CAV'16)*.

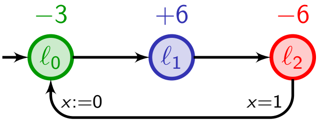
Further problems: Energy management

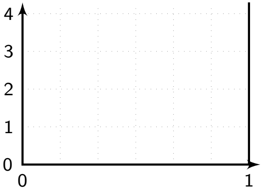
Example



Further problems: Energy management

Example



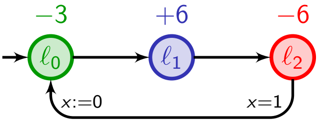


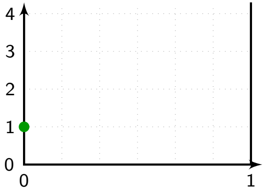
"energy is $\geq_0$"

- Lower-bound problem (**L**)

Further problems: Energy management

Example





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Further problems: Energy management

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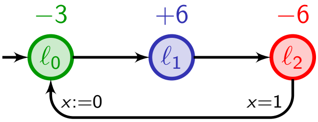

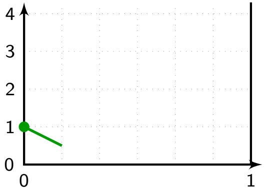

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Further problems: Energy management

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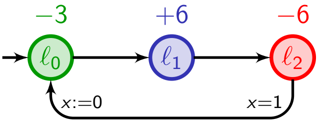

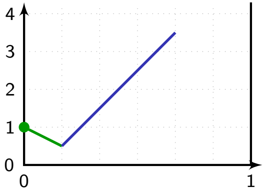

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Further problems: Energy management

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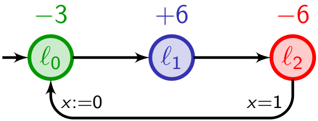

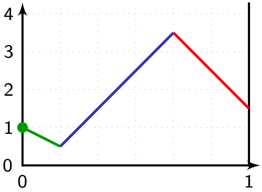

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Further problems: Energy management

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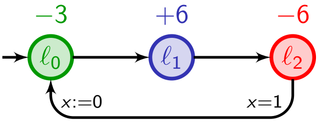
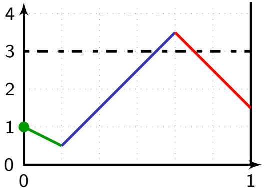




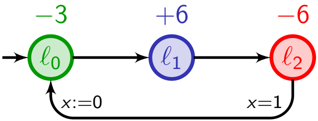
"energy is in [0,3]"

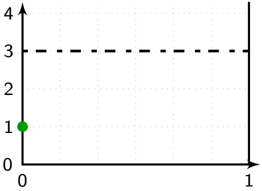
- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Skip energy management

Further problems: Energy management

Example





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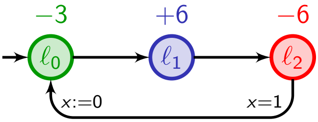
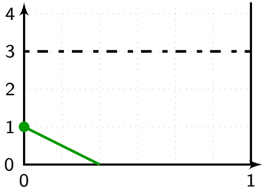
Further problems: Energy management

Example





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- Lower-bound problem (**L**)
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Skip energy management

Further problems: Energy management

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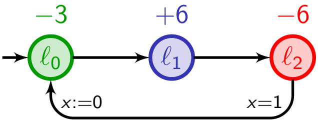

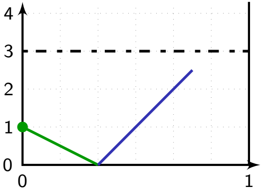

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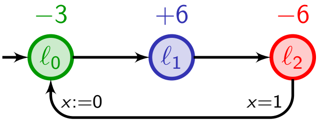
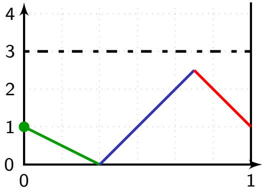
Further problems: Energy management

Example





"energy is in [0,3]"

- Lower-bound problem (**L**)
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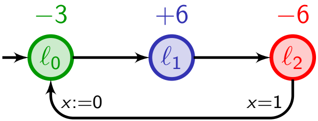
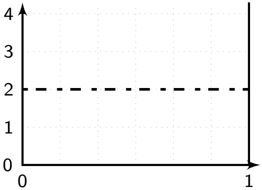
Further problems: Energy management

Example



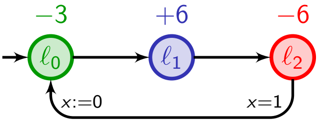


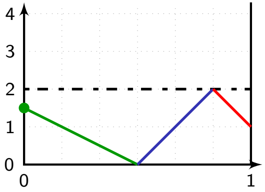
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example





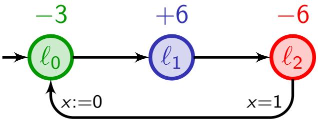
"energy is in [0,2]"

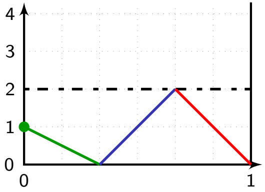
- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Skip energy management

Further problems: Energy management

Example



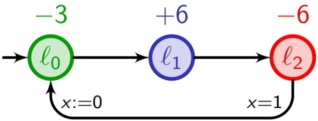


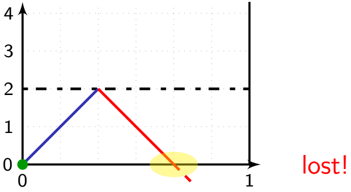
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



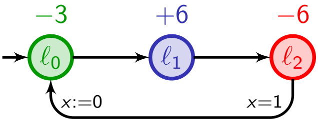


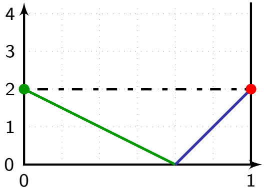
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



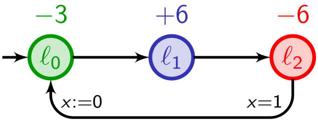


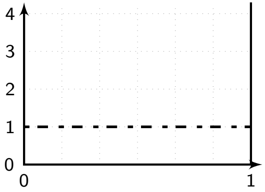
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example





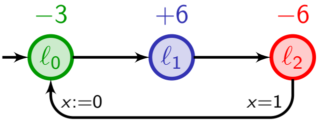
"energy is in [0,1]"

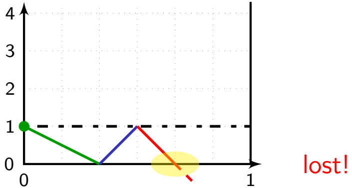
- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Skip energy management

Further problems: Energy management

Example





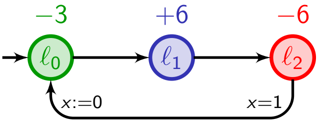
"energy is in [0,1]"

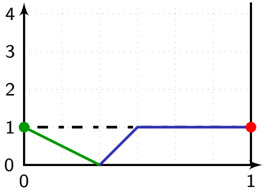
- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Skip energy management

Further problems: Energy management

Example





"energy is in $[0,1]$ with a weak upper bound"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)
- Lower-and-weak-upper-bound problem (**L+W**)

The **L**-problem: use the corner-point abstraction?

Idea: delay in the most profitable location

$\rightsquigarrow$ the corner-point abstraction

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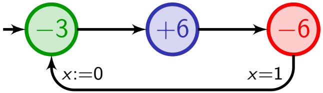
Example



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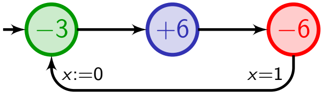

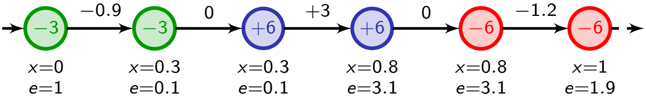

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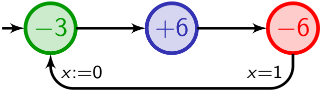

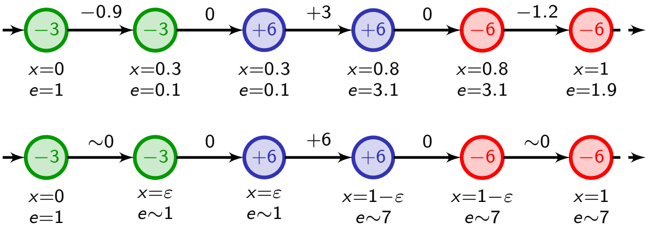

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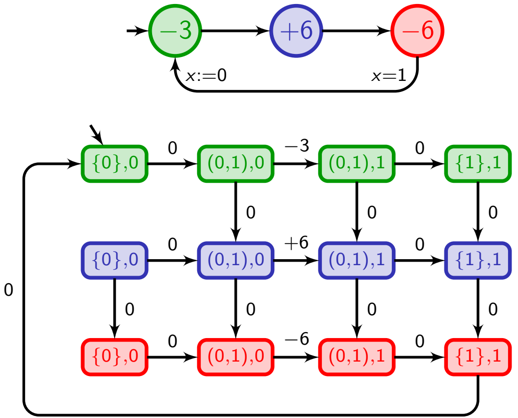


The **L**-problem: use the corner-point abstraction?

Idea: delay in the most profitable location

$\rightsquigarrow$ the corner-point abstraction

Theorem [BFLMS08]

The corner-point abstraction is sound and complete for single-clock WTA with no discrete costs. Hence the existential **L**-problem is in PTIME in that case.

[BFLMS08] Bouyer, Fahrenberg, Larsen, Markey, Srba. Infinite runs in weighted timed automata with energy constraints *(FORMATS'08)*.

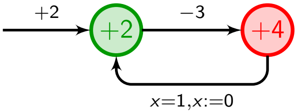
The **L**-problem: use the corner-point abstraction?

Idea: delay in the most profitable location

$\rightsquigarrow$ the corner-point abstraction

Remark

The corner-point abstraction is not correct with discrete costs.



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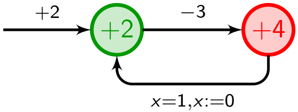

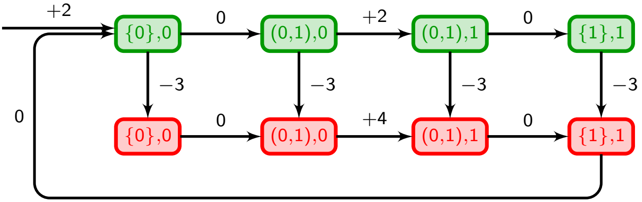

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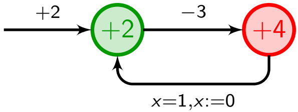

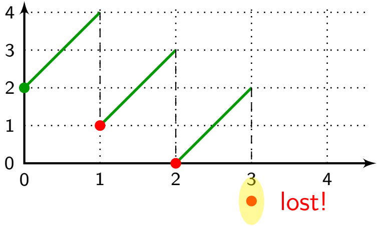

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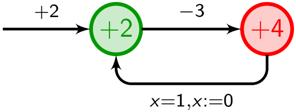

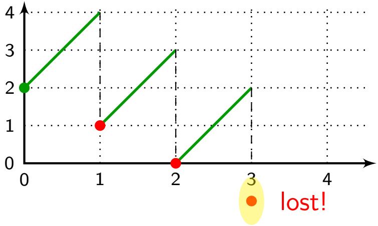

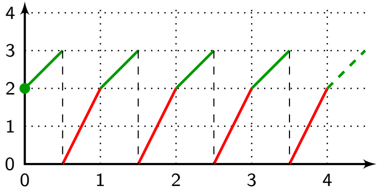

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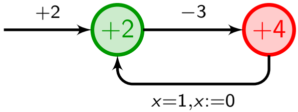
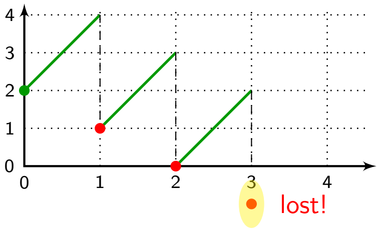
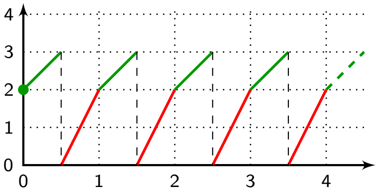


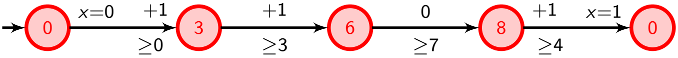




$\rightsquigarrow$ requires new developments!

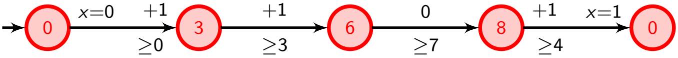
The **L**-problem: computing optimal delays

Example



The **L**-problem: computing optimal delays

Example

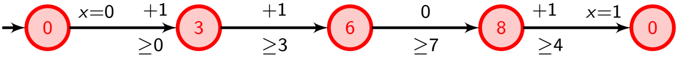


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example

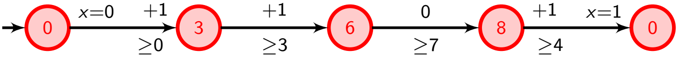


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—				—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—			0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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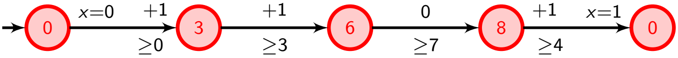
The **L**-problem: computing optimal delays

Example

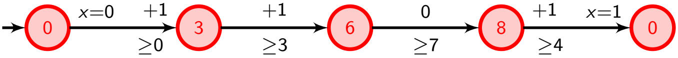


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—		$\frac{1}{2}$	0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

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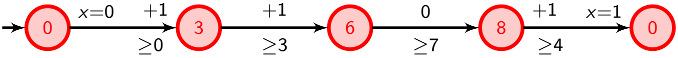


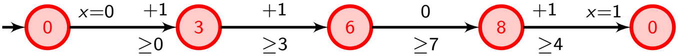
t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

minimal initial credit required: $\frac{1}{2}$, yields final credit 8.

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

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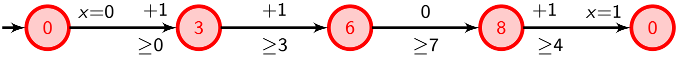

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit
$\frac{1}{2} + \delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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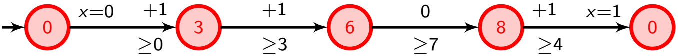

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit $\frac{1}{2}+\delta$		$\frac{1}{2}-\frac{\delta}{3}$			

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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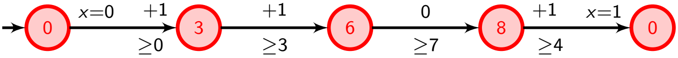

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit $\frac{1}{2}+\delta$		$\frac{1}{2}-\frac{\delta}{3}$	$\frac{1}{2}$		

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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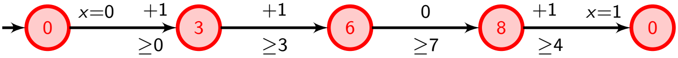

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit $\frac{1}{2}+\delta$		$\frac{1}{2}-\frac{\delta}{3}$	$\frac{1}{2}$	$\frac{\delta}{3}$	

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The **L**-problem: computing optimal delays

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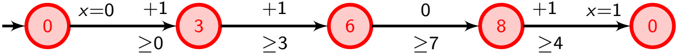

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit $\frac{1}{2}+\delta$	$\frac{1}{2}-\frac{\delta}{3}$	$\frac{1}{2}$	$\frac{\delta}{3}$	final credit $8+\frac{8}{3}\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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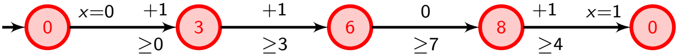
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit 2	0	$\frac{1}{2}$	$\frac{1}{2}$	final credit 12

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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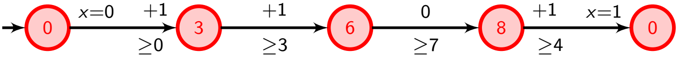
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit $2+\delta$ 0 $\frac{1}{2}-\frac{\delta}{6}$ $\frac{1}{2}+\frac{\delta}{6}$ final credit $12+\frac{8}{6}\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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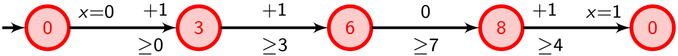


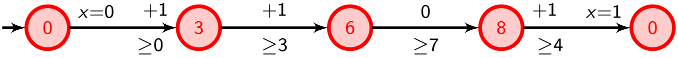
t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit 5	0	0	1	final credit 16

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit $5+\delta$	0	0	1	final credit $16+\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

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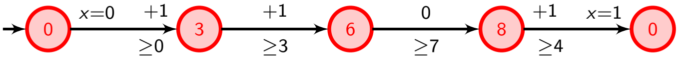

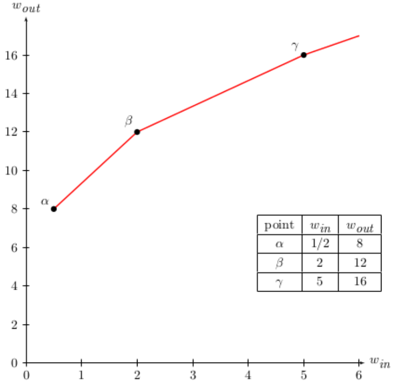

point	w_{in}	w_{out}
α	1/2	8
β	2	12
γ	5	16

The **L**-problem: concluding

Theorem

Optimization, reachability and existence of infinite runs satisfying the constraint ≥ 0 can be decided in EXPTIME in single-clock WTA

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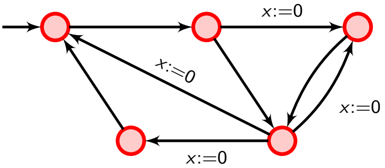
Optimization, reachability and existence of infinite runs satisfying the constraint ≥ 0 can be decided in EXPTIME in single-clock WTA

- transform the automaton into an automaton with energy functions;



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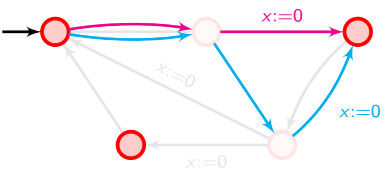

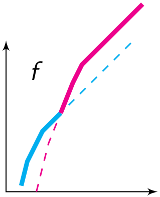

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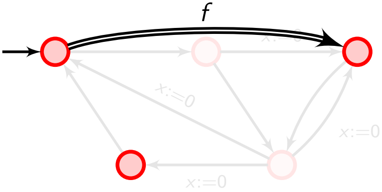

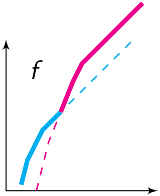

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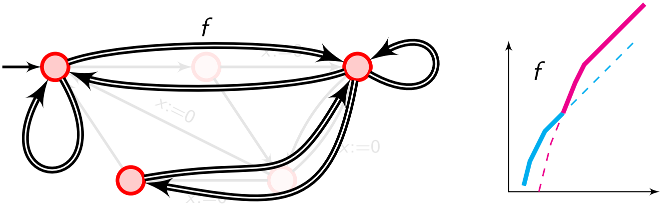

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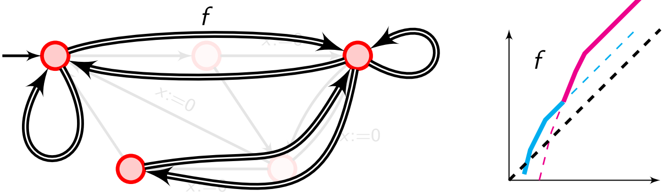


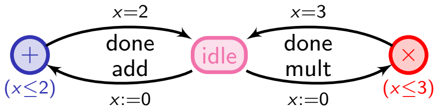
- check if simple cycles can be iterated (or if a Zeno cycle can be reached...)

Outline

Why (timed) games?

- to model uncertainty

Example of a processor in the taskgraph example



Why (timed) games?

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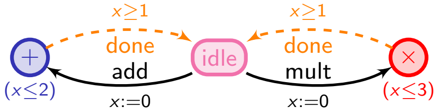
Example of a processor in the taskgraph example



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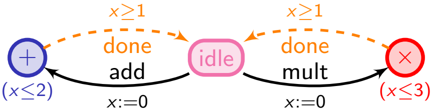
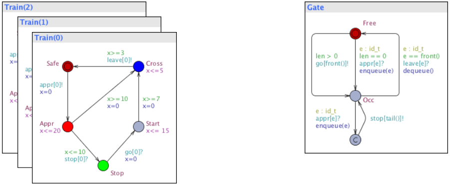


- to model an interaction with the environment

Example of the gate in the train/gate example



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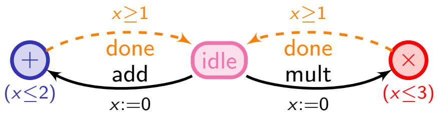

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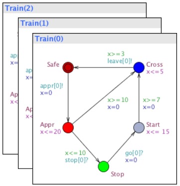

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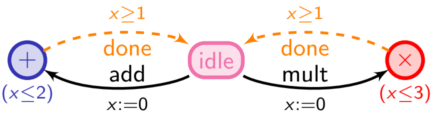

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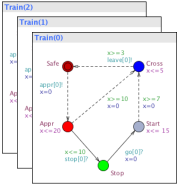
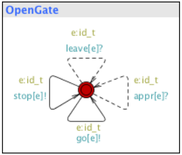
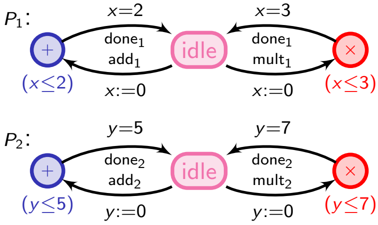


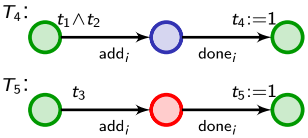


Modelling the task graph scheduling problem

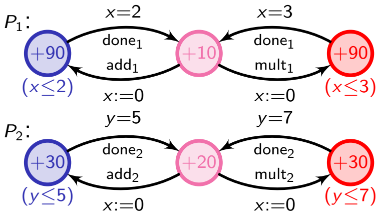
- Processors



- Tasks



- Modelling energy



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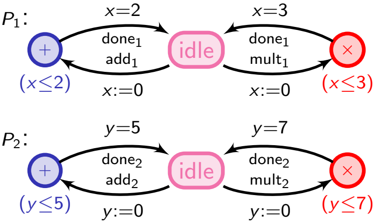

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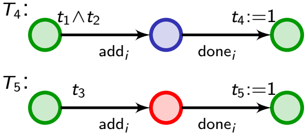

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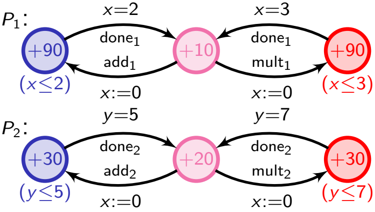
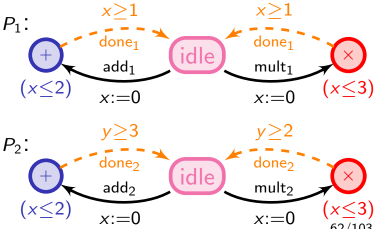


- Modelling uncertainty



Modelling the task graph scheduling problem

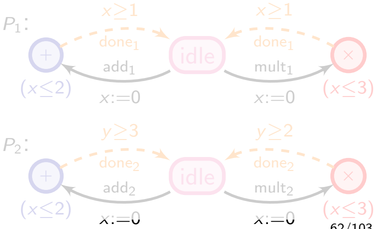
- Processors

- Tasks

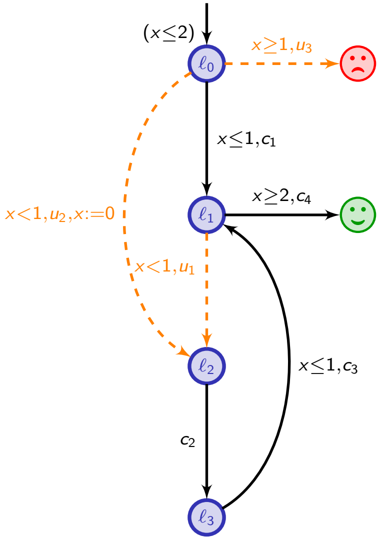
A (good) schedule is a strategy in the product game (with a low cost)

- Modelling energy

- Modelling uncertainty



An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺

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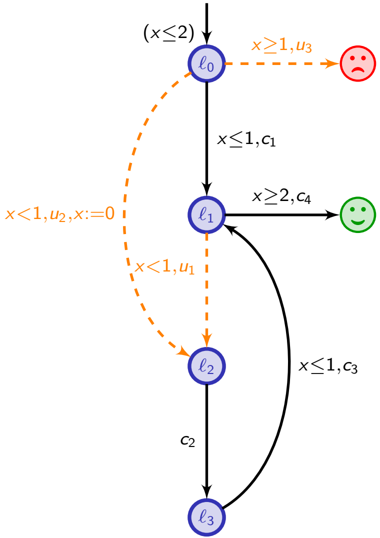

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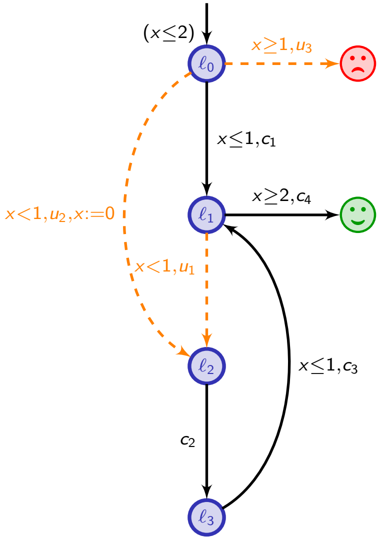

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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

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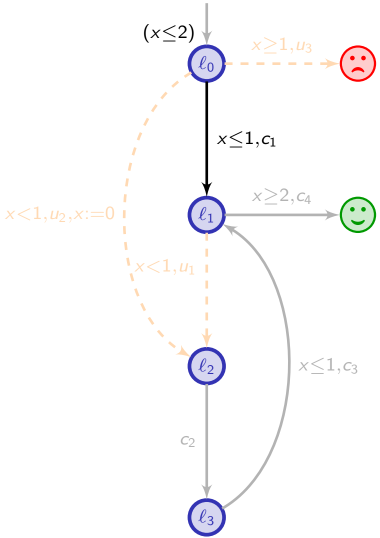

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A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$

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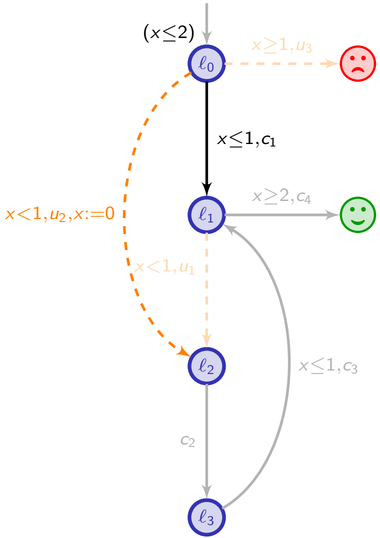

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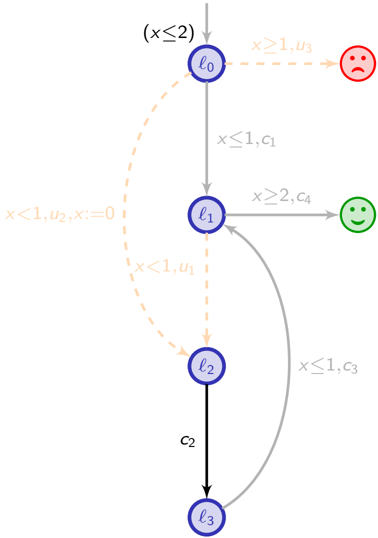

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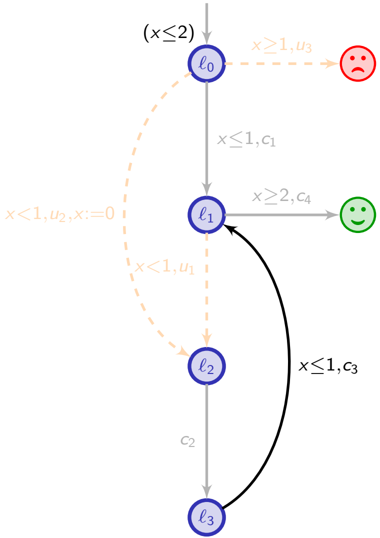
  $\rightsquigarrow$ can be preempted by u_2
- from $(\ell_2, \star)$, play $(1-\star, c_2)$

An example of a timed game



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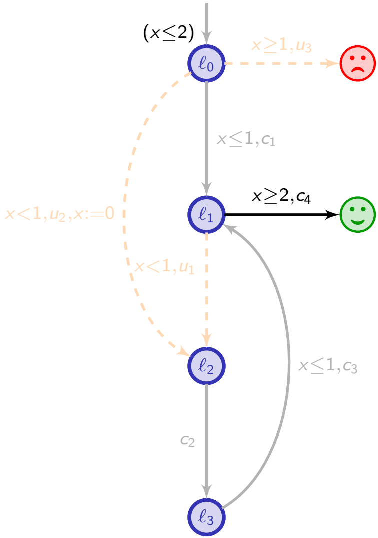
$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$
  $\rightsquigarrow$ can be preempted by u_2
- from $(\ell_2, \star)$, play $(1 - \star, c_2)$
- from $(\ell_3, 1)$, play $(0, c_3)$

An example of a timed game



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A (memoryless) winning strategy

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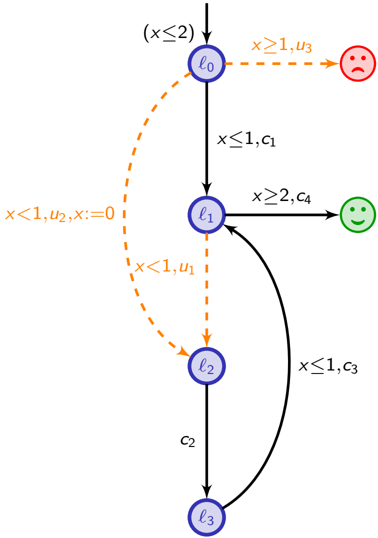
  $\rightsquigarrow$ can be preempted by u_2
- from $(\ell_2, \star)$, play $(1 - \star, c_2)$
- from $(\ell_3, 1)$, play $(0, c_3)$
- from $(\ell_1, 1)$, play $(1, c_4)$

An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺
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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

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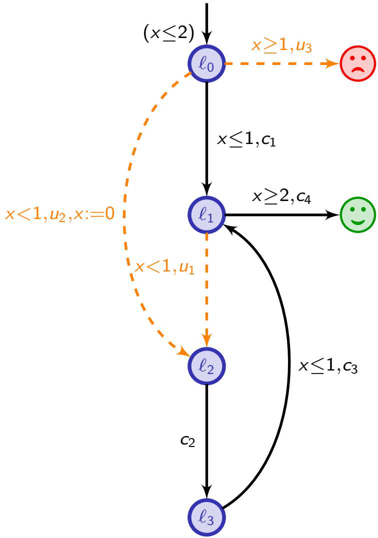

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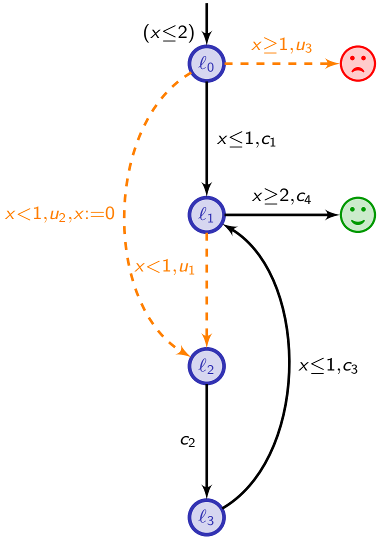

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- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and “region-based” strategies are sufficient.

$\leadsto$ classical regions are sufficient for solving such problems
a region-closed attractor can be computed

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
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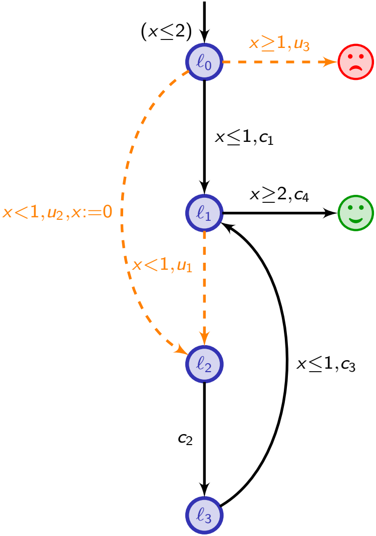
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Back to the example: computing winning states



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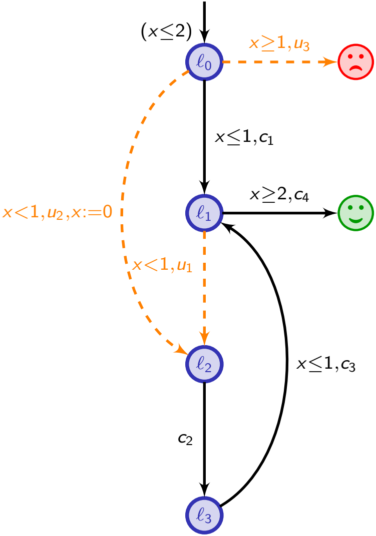

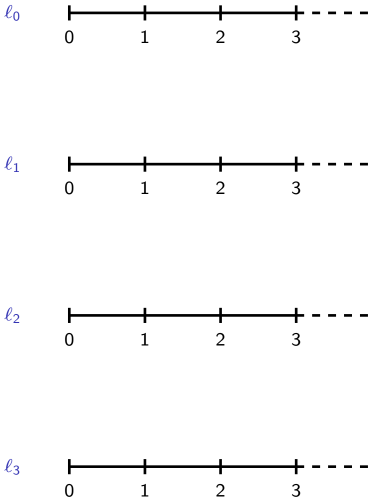

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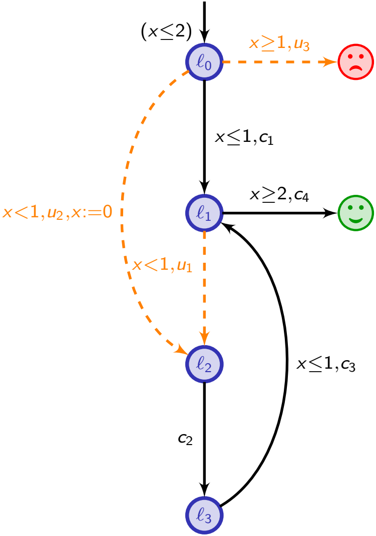

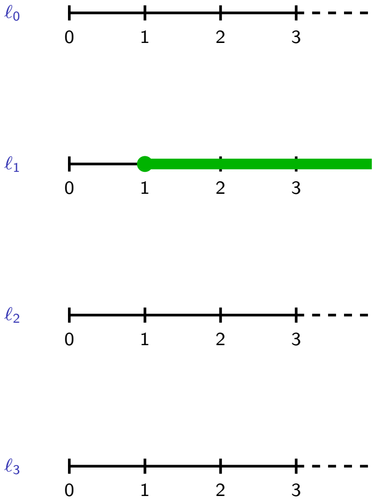

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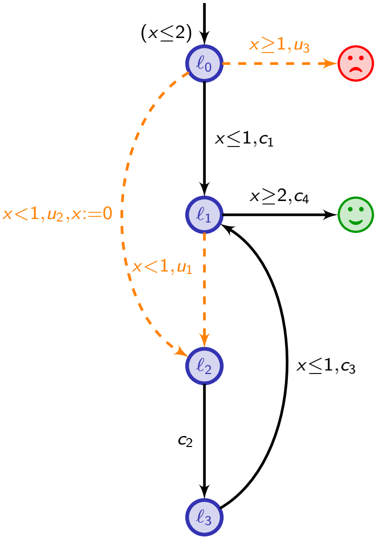

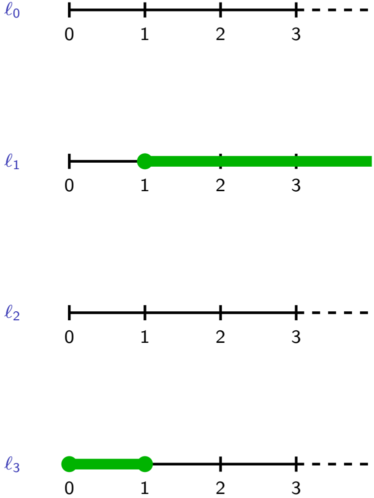

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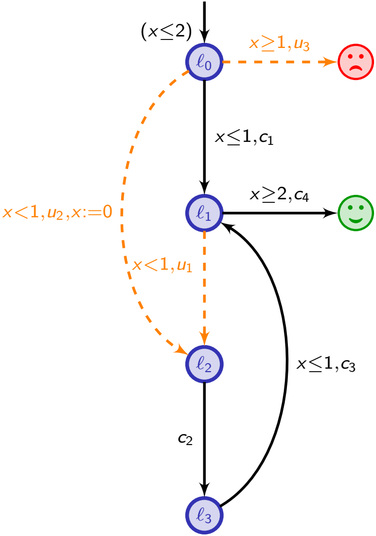

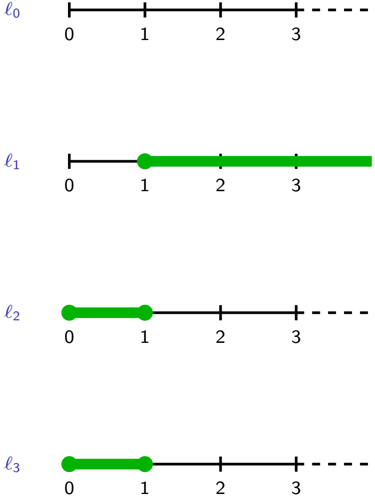

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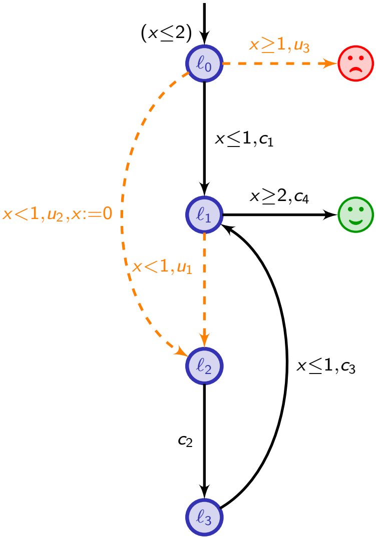

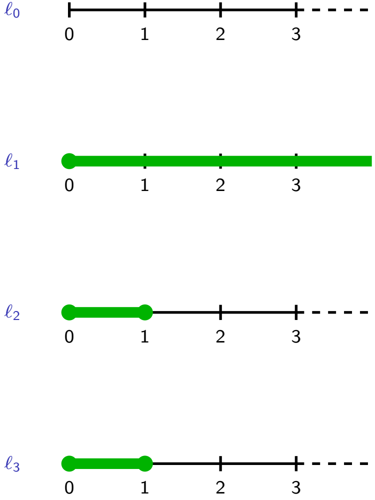

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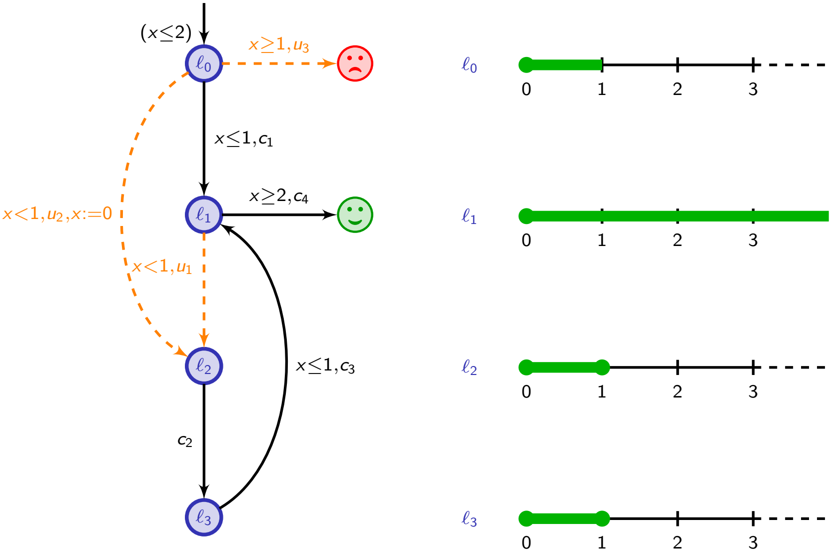

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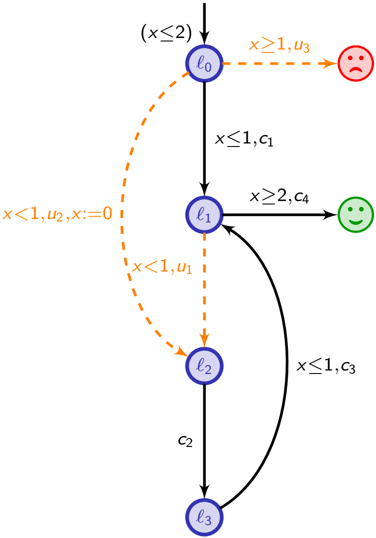

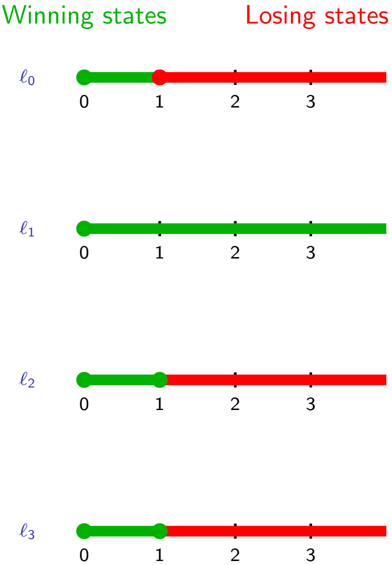

Decidability *via* attractors

Skip attractors

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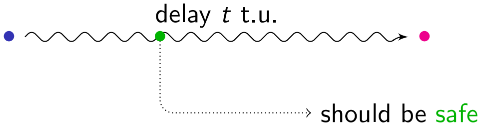

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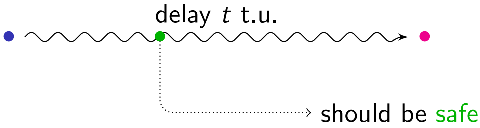

$$\mathrm{Pred}_\delta(X, \mathrm{Safe}) = \{\bullet \mid \exists t \geq 0,\ \bullet \xrightarrow{\delta(t)} \bullet$$
$$\text{and } \forall 0 \leq t' \leq t,\ \bullet \xrightarrow{\delta(t')} \bullet \in \mathrm{Safe}\}$$

Timed games with a reachability objective

We write:

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Stability w.r.t. regions

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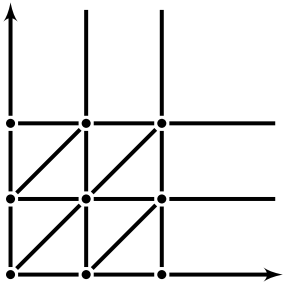

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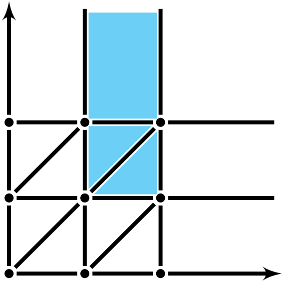

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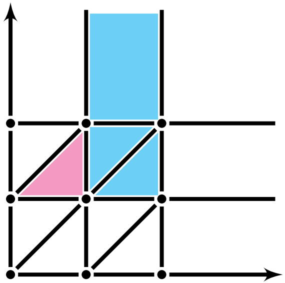

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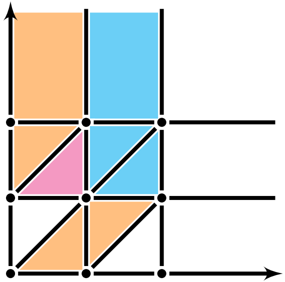

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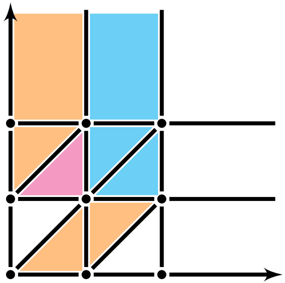

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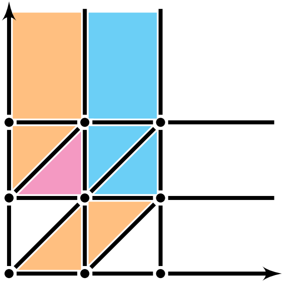

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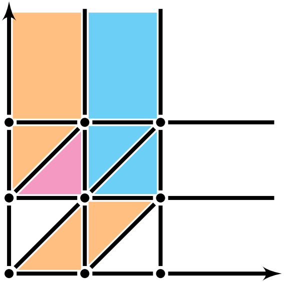

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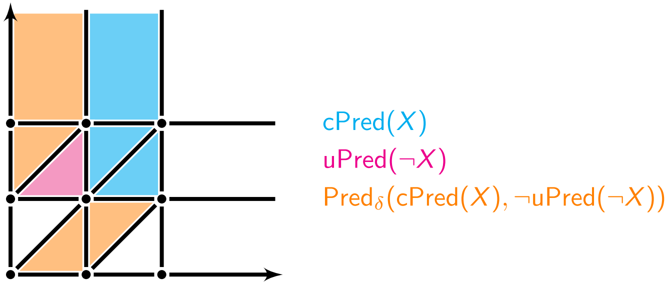

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... and is correct

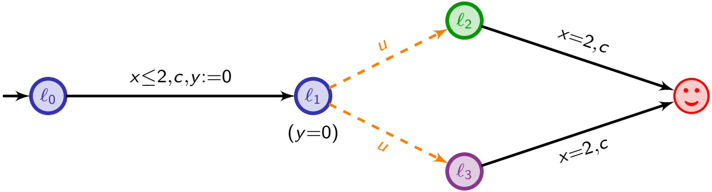
And in practice?

- A zone-based forward algorithm with backtracking [CDF+05,BCD+07]

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient On-the-Fly Algorithms for the Analysis of Timed Games *(CONCUR'05)*.
[BCD+07] Behrmann, Cougnard, David, Fleury, Larsen, Lime. UPPAAL-Tiga: Time for Playing Games! *(CAV'07)*.

Outline

A simple timed game



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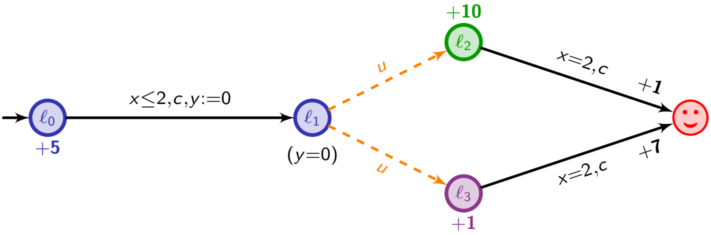

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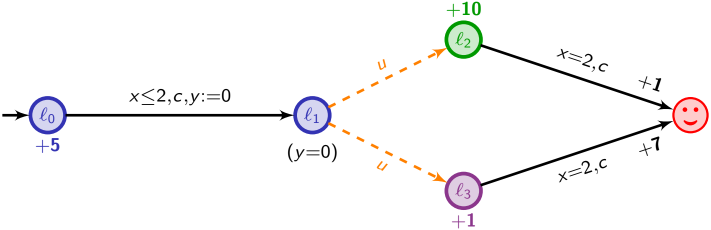
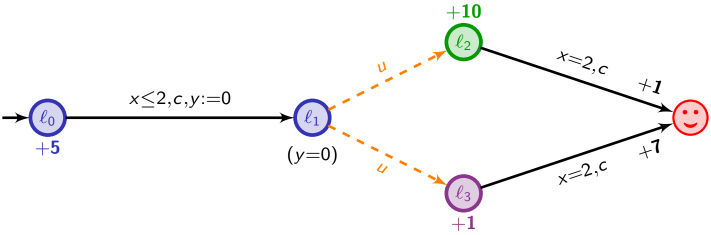


Question: what is the optimal cost we can ensure while reaching ?

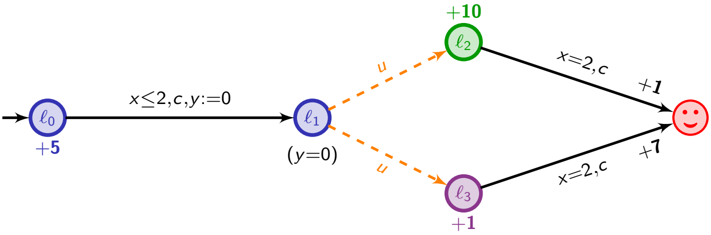
A simple weighted timed game



Question: what is the optimal cost we can ensure while reaching ☺?

$5t + 10(2 - t) + 1$

A simple weighted timed game



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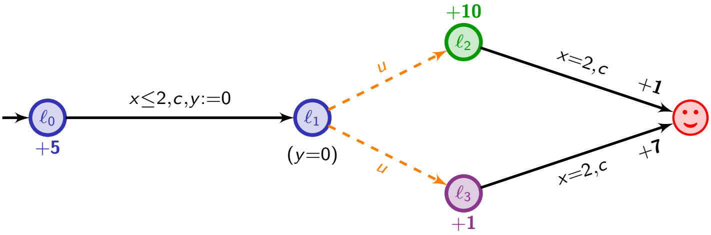
$$5t + 10(2-t) + 1 \ , \ 5t + (2-t) + 7$$

A simple weighted timed game



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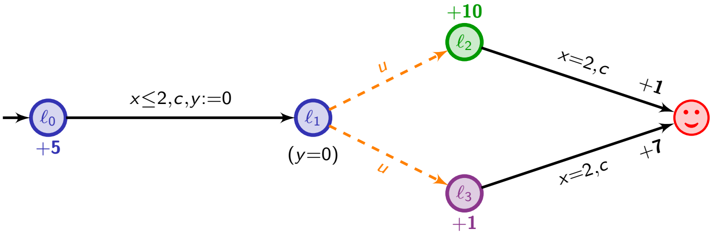

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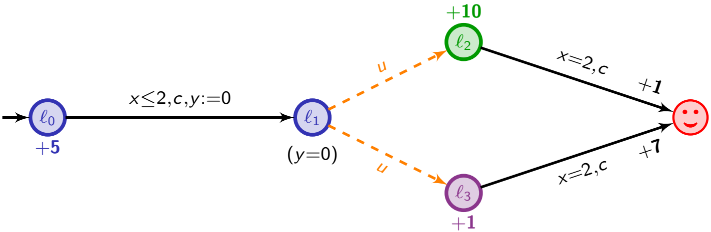
$$\inf_{0 \leq t \leq 2} \max \left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

A simple weighted timed game



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$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t=\frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games (1)

This topic has been fairly hot these since the 2000's
[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11]
[HIM13,BGK+14,BJM15,BMR17,BMR18,MPR20]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.
[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.
[Rut11] Rutkowski. Two-player reachability-price games on single-clock timed automata *(QAPL'11)*.
[HIM13] Hansen, Ibsen-Jensen, Miltersen. A faster algorithm for solving one-clock priced timed games *(CONCUR'13)*.
[BGK+14] Brihaye, Geeraerts, Krishna, Manasa, Monmege, Trivedi. Adding Negative Prices to Priced Timed Games *(CONCUR'14)*.
[BJM15] Bouyer, Jaziri, Markey. On the value problem in weighted timed games *(CONCUR'15)*.
[BMR17] Busatto-Gaston, Monmege, Reynier. Optimal Reachability in Divergent Weighted Timed Games *(FoSSaCS'17)*.
[BMR18] Busatto-Gaston, Monmege, Reynier. Symbolic Approximation of Weighted Timed Games *(FSTTCS'18)*.
[MPR20] Monmege, Parreaux, Reynier. Reaching Your Goal Optimally by Playing at Random with No Memory *(CONCUR'20)*.

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[LMM02]

Tree-like weighted timed games can be solved in 2EXPTIME.

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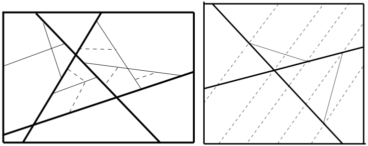
Tree-like weighted timed games can be solved in 2EXPTIME.

[ABM04,BCFL04]

Depth-k weighted timed games can be solved in EXPTIME. There is a symbolic algorithm to solve weighted timed games **with a strongly non-Zeno cost**.



Optimal reachability in weighted timed games (2)

[BBR05,BBM06,BJM15]

In weighted timed games, the optimal cost (and the value) cannot be computed, as soon as games have three clocks or more.

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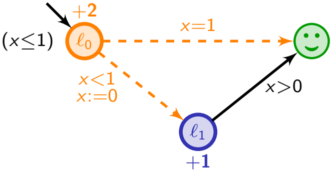
[BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (resp. with two rates). They are PTIME-hard.

What is easier with a single clock?

- Memoryless strategies can be non-optimal...



[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.

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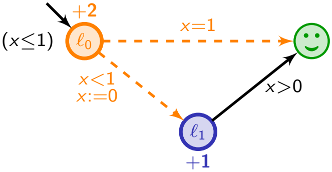

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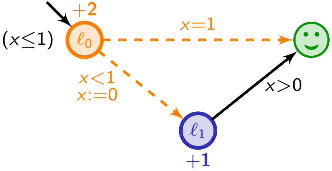

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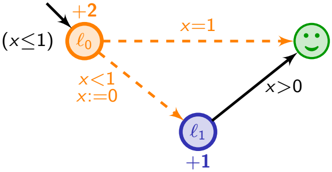

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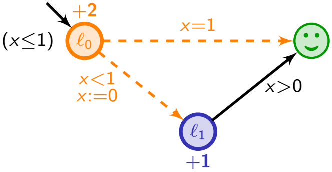

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- Rather involved proofs of correctness

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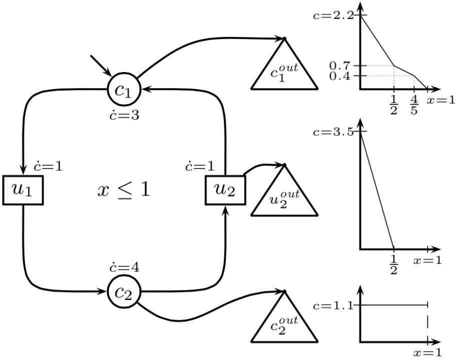

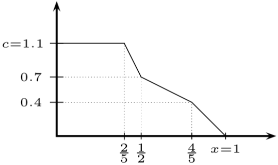

$$\sigma(c_2, x) = \begin{cases} c_2^{out} & \text{if } 0 \leq x < 2/5 \\ c_2 & \text{if } 2/5 \leq x < 1/2 \\ u_2 & \text{if } 1/2 \leq x \leq 1 \end{cases}$$

Computing the optimal cost: why is that hard?

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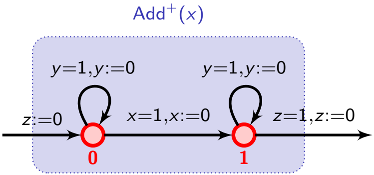
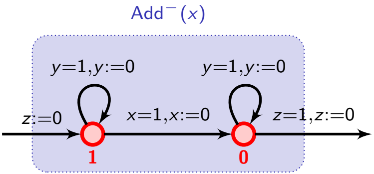


The cost is increased by x_0



The cost is increased by $1-x_0$

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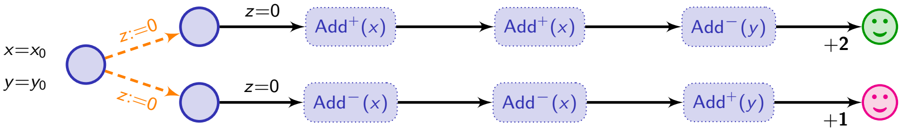

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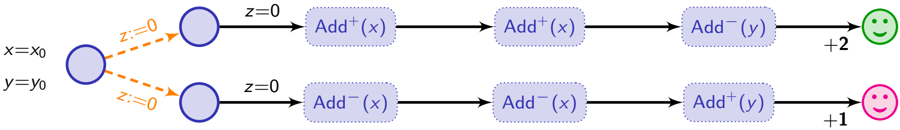

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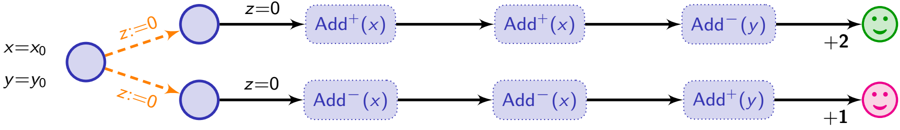

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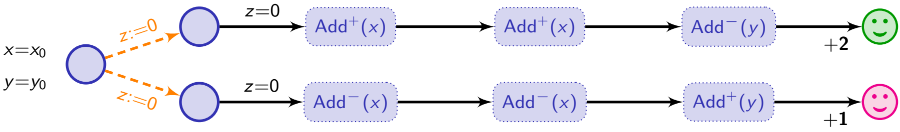

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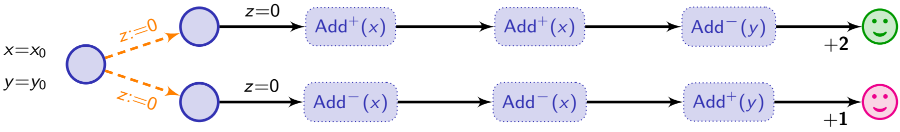

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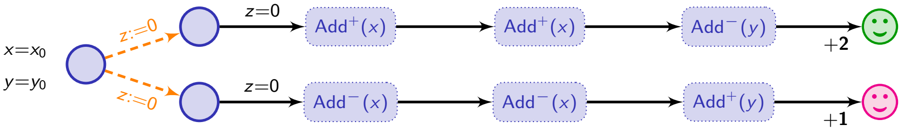

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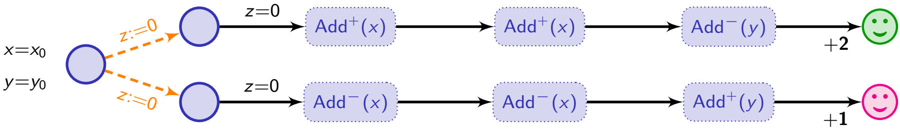

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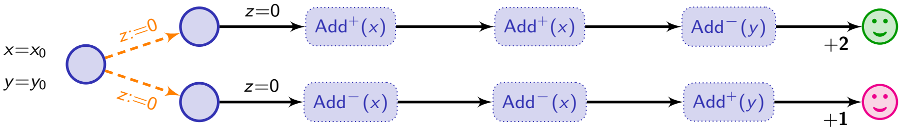

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- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

Computing the optimal cost: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the counter values c_1 and c_2 are encoded by two clocks:

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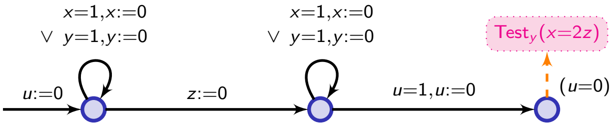
Globally, $(x \leq 1, y \leq 1, u \leq 1)$



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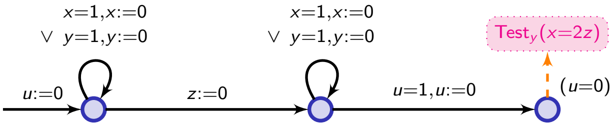

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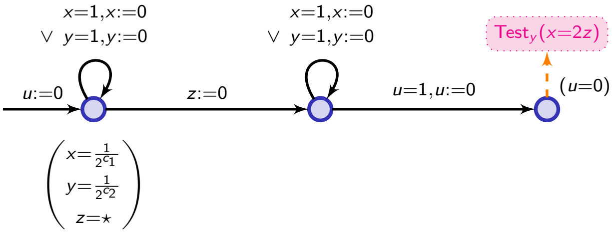

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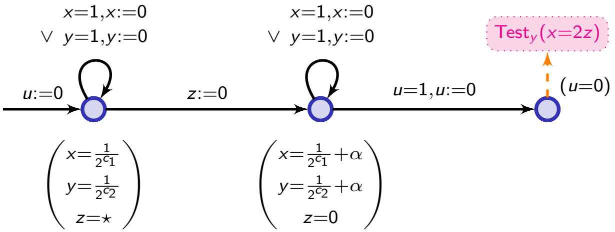

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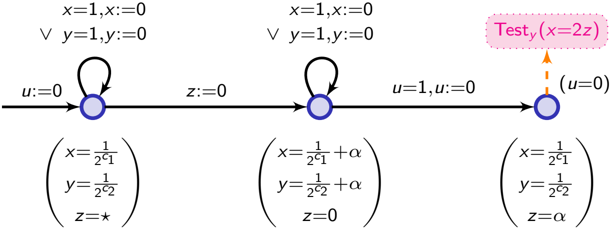

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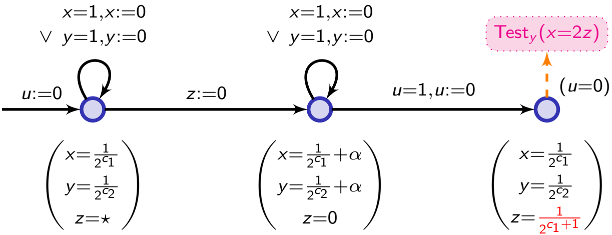
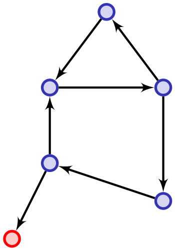


Shape of the reduction



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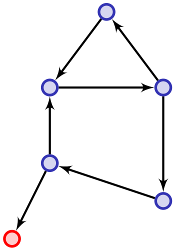

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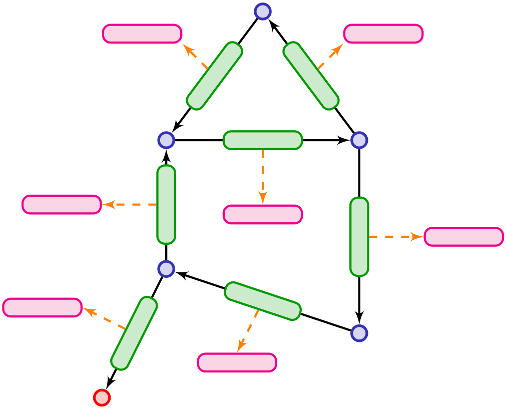
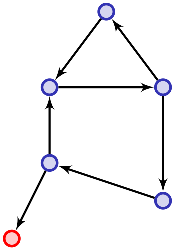


Instruction

Test module (acyclic)

Shape of the reduction



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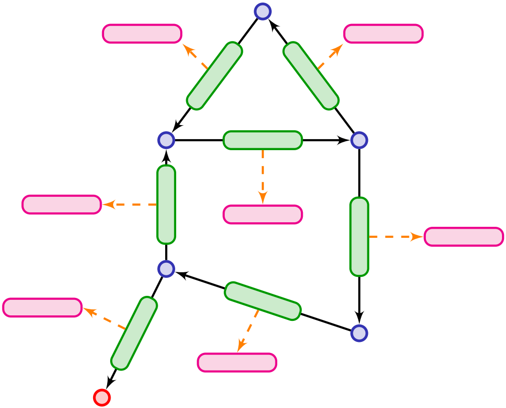

Instruction

Test module (acyclic)

Cost 0 within the core of the game

Some further subtlety

Skip

Value of the game = infimum of all costs of strategies

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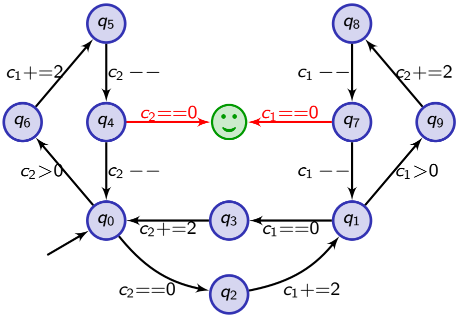
The value of the game is 3, but no strategy has cost 3.



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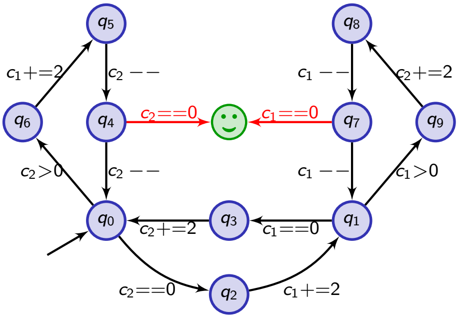
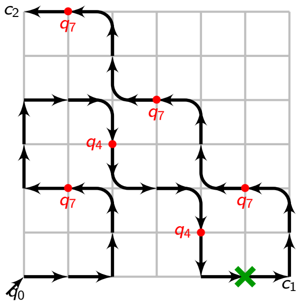
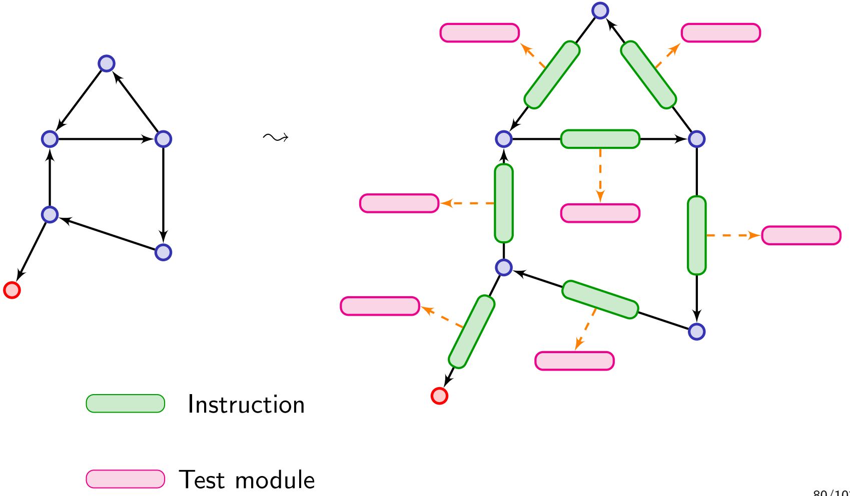




A snapshot on the undecidability proof



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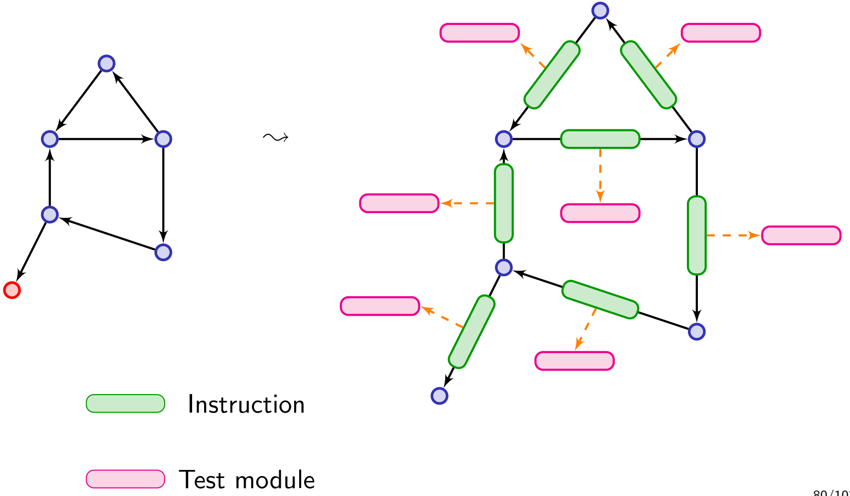

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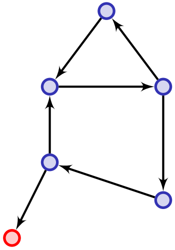

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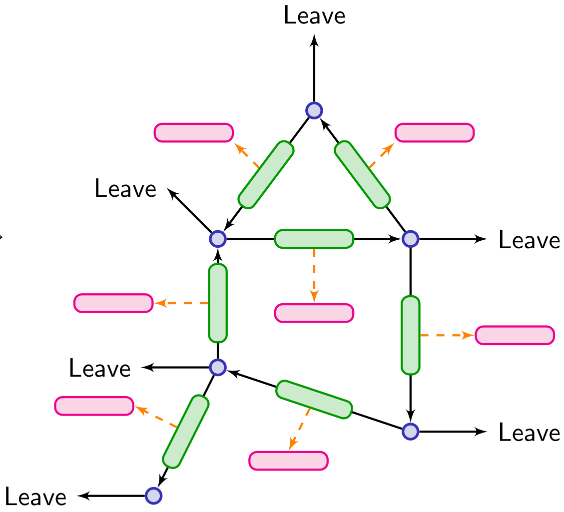

Leave with cost $3 + 1/2^n$ (n: length of the path)

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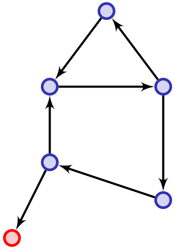
$\mathcal{M}$ does not halt iff the value of $\mathcal{G}_{\mathcal{M}}$ is 3



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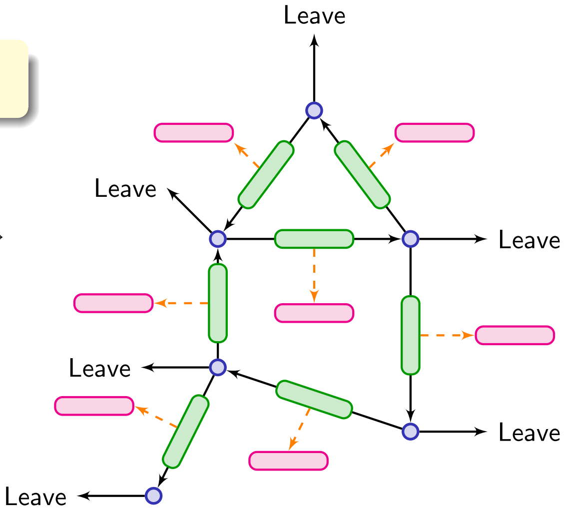

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Optimal cost is computable...

... when cost is strongly non-zeno. [AM04,BCFL04]

There is $\kappa > 0$ s.t. for every region cycle C, for every real run ϱ read on C,

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- Almost-optimality in practice should be sufficient
- Even when we know how to compute the value, we are only able to synthesize almost-optimal strategies...

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Approximation of the optimal cost

Theorem

Let $\mathcal{G}$ be a weighted timed game, in which the cost is almost-strongly non-zeno. For every $\epsilon > 0$, one can compute:

- two values v_ϵ^- and v_ϵ^+ such that

$$|v_\epsilon^+ - v_\epsilon^-| < \epsilon \quad \text{and} \quad v_\epsilon^- \leq \text{optcost}_{\mathcal{G}} \leq v_\epsilon^+$$

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Skip approximation scheme

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$$|v_\epsilon^+ - v_\epsilon^-| < \epsilon \quad \text{and} \quad v_\epsilon^- \leq \mathsf{optcost}_{\mathcal{G}} \leq v_\epsilon^+$$

- one strategy σ_ϵ such that

$$\mathsf{optcost}_{\mathcal{G}} \leq \mathsf{cost}(\sigma_\epsilon) \leq \mathsf{optcost}_{\mathcal{G}} + \epsilon$$

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- Standard technics: unfold the game to get more precision, and compute two adjacency sequences

Approximation of the optimal cost

Theorem

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- Standard technics: unfold the game to get more precision, and compute two adjacency sequences

$\rightsquigarrow$ This is not possible here

There might be runs with prefixes of arbitrary length and cost 0 (e.g. the game of the undecidability proof)

Idea for approximation

Idea

Only partially unfold the game:

- Keep components with cost 0 untouched – we call it the kernel
- Unfold the rest of the game

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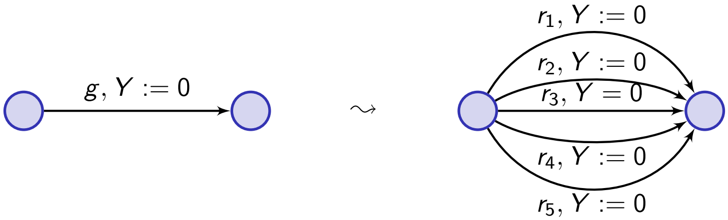
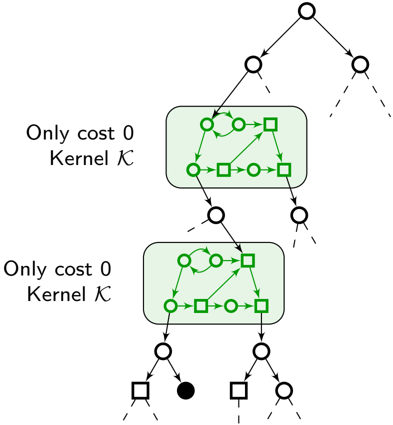


Idea of the proof: Semi-unfolding



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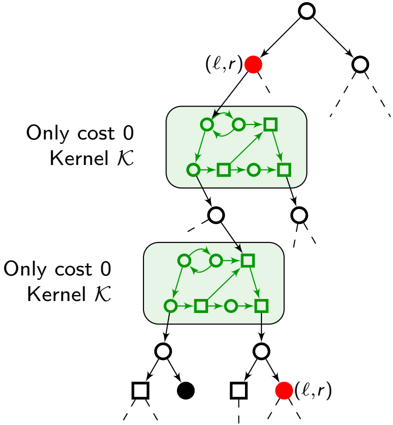

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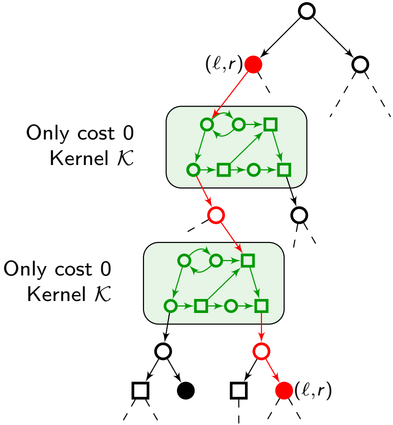

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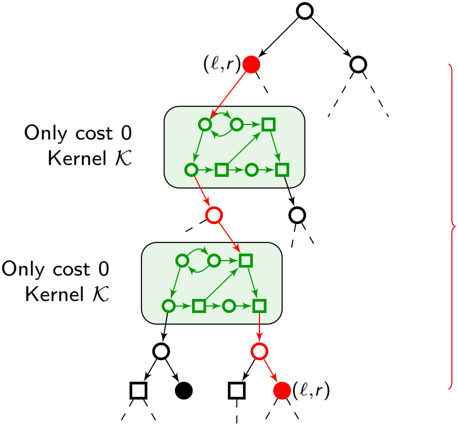

Hypothesis:
cost > 0 implies cost $\geq \kappa$

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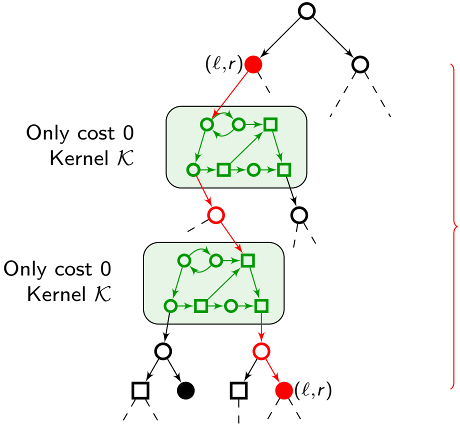
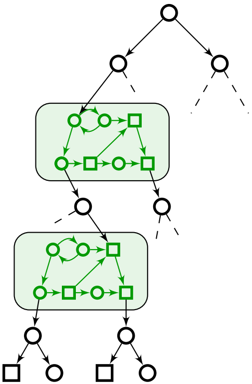


Hypothesis:
cost > 0 implies cost $\geq \kappa$

Conclusion: we can stop unfolding the game after finitely many steps

Approximation scheme



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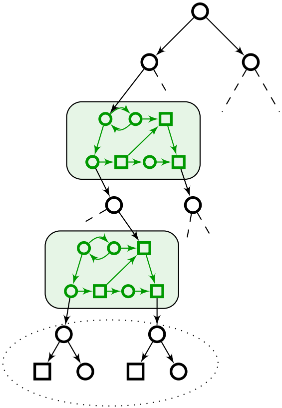

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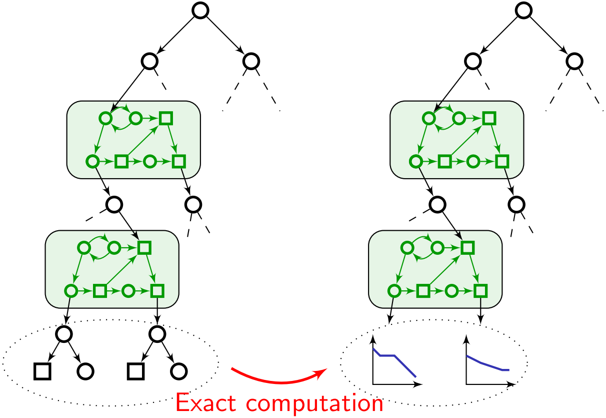

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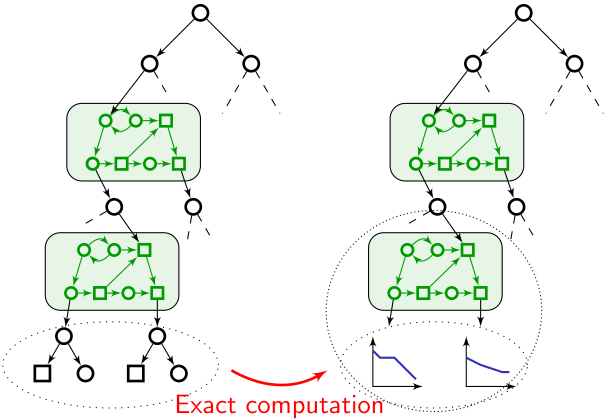

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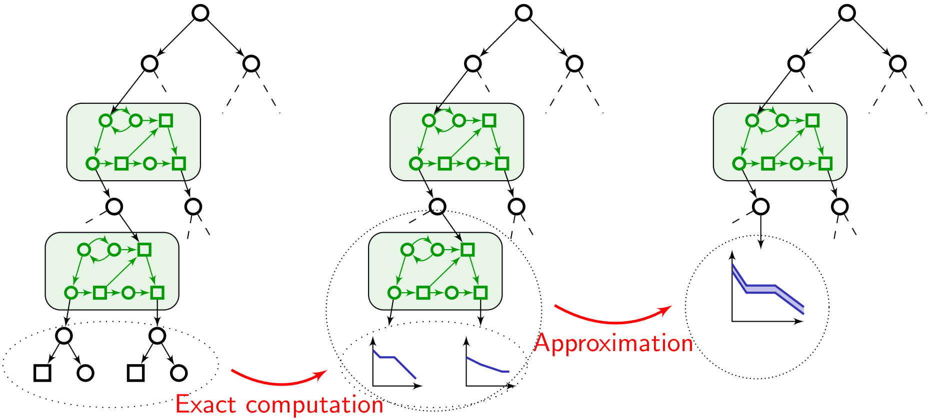

First step: Tree-like parts

$\rightsquigarrow$ Goes back to [LMM02]

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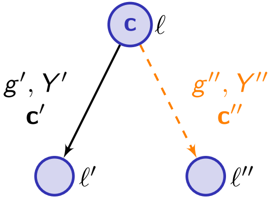

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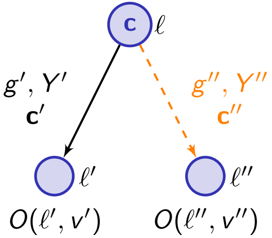

$O(\ell, v) =$

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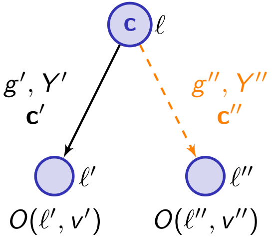

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'}$$

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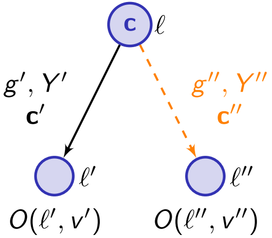

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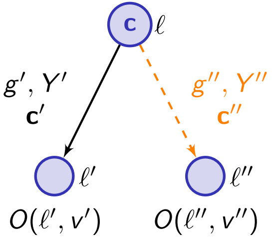

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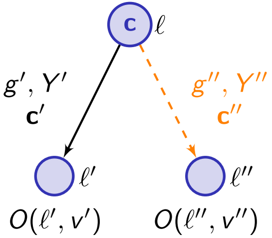

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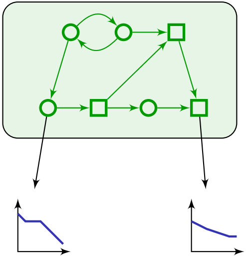

Output cost functions f

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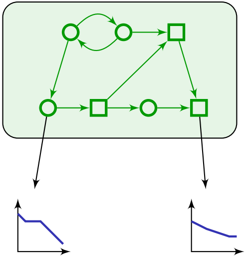

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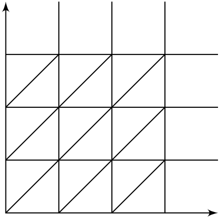

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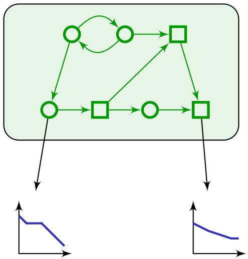

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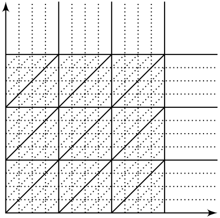

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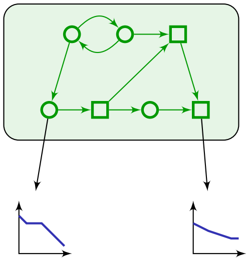

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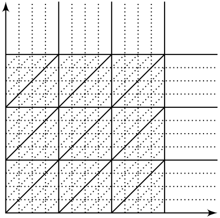

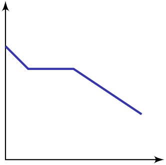

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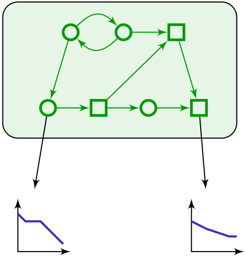

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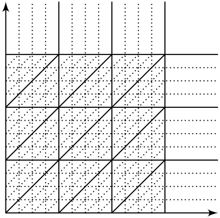
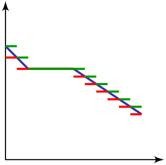
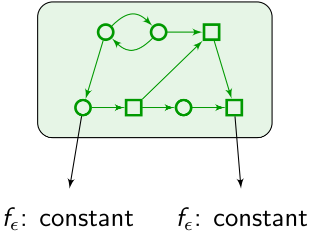


2. Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+



Second step: Kernels

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ



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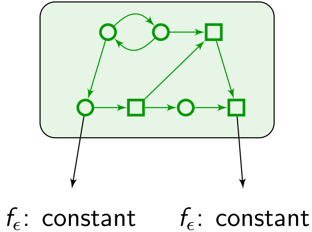

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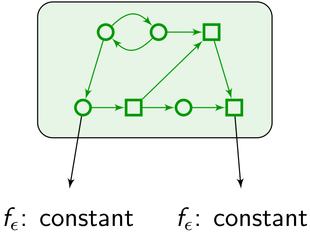

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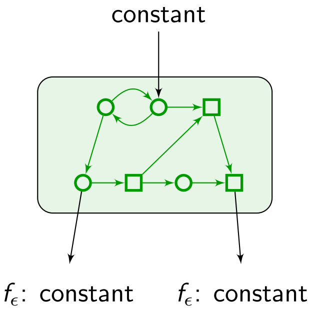

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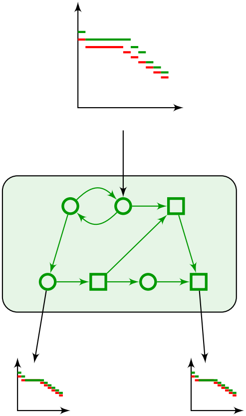

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$\rightsquigarrow$ We have computed ϵ-approximations of the optimal cost, which are constant within small regions. Corresponding strategies can be inferred

Outline

Tools for (weighted) timed automata and games

- Many tools and prototypes everywhere on earth...

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- Tool-suite Uppaal, developed in Aalborg (Denmark) and originally Uppsala (Sweden) since 1995
  - Uppaal for timed automata
  - Uppaal-TiGa for timed games
  - Uppaal-Cora for weighted timed automata

Uppaal url: http://www.uppaal.org

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- The new tool Tchecker, developed within ANR Ticktac project mostly by Frédéric Herbreteau (LaBRI), Ocan Sankur (IRISA), Gérald Point (LaBRI), Philipp Schlehuber-Caissier (LRDE) and Alexandre Duret-Lutz (LRDE)

`http://www.irisa.fr/sumo/ticktac/`

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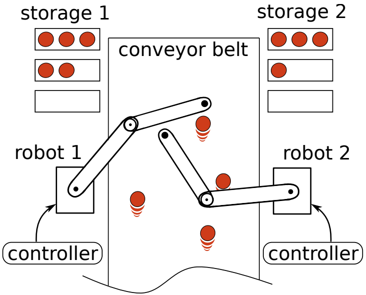
TiAMo url: `https://git.lsv.fr/colange/tiamo`
Tchecker url: `https://github.com/ticktac-project/tchecker`

Outline

Very refreshing collaboration with Nicolas Markey (LSV at that time, now at IRISA), Nicolas Perrin (ISIR) and Philipp Schlehuber-Caissier (ISIR at that time, now at LRDE)

Example problem, objective and approach



[BMPS15] Bouyer, Markey, Perrin, Schlehuber-Caissier. Timed-Automata Abstraction of Switched Dynamical Systems Using Control Funnels *(FORMATS'15)*.
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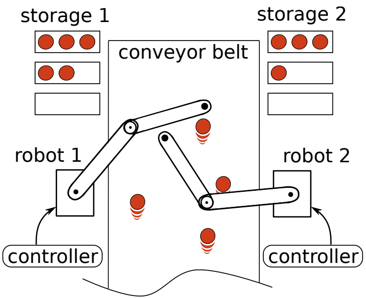

- Infinitely many configurations
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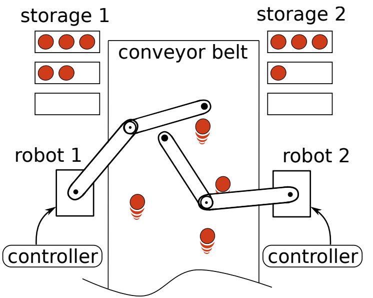

Goal: Synthesize a controller:

- Which robot handles an object
- How to avoid collision
- Don't miss any object

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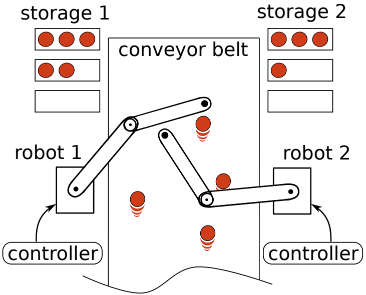

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Approach:

- Discretization of the behaviour via a fixed set of continuous controllers
- Create an abstraction and use previous results

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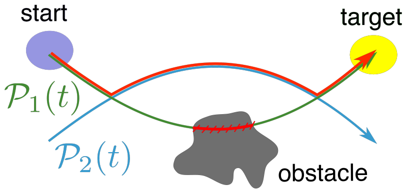
Our approach

Simplistic idea: fixed set of reference trajectories + property



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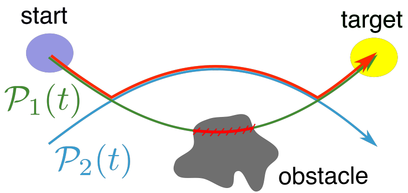
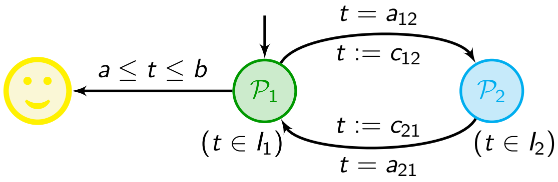
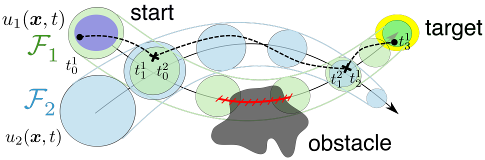


Corresponding timed automaton:



Our approach

More realistic idea: fixed set of funnels for control law + property



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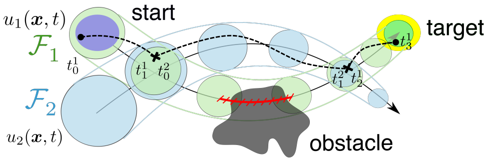

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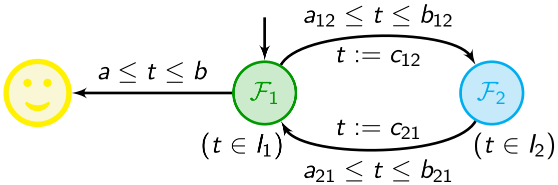
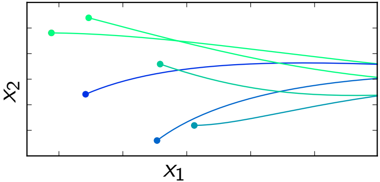


Control funnels

System with continuous dynamics $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, t)$



Control funnels

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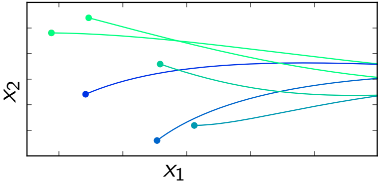

A (control) funnel is a trajectory $\mathcal{F}(t)$ of a set in the state space such that, for any trajectory $\boldsymbol{x}(t)$ of the dynamical system:

$$\forall t_0 \in \mathbb{R},\ \boldsymbol{x}(t_0) \in \mathcal{F}(t_0) \Rightarrow \forall t \geq t_0,\ x(t) \in \mathcal{F}(t)$$

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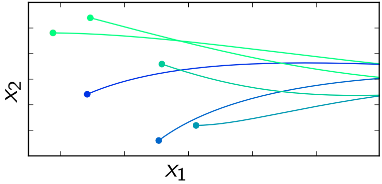

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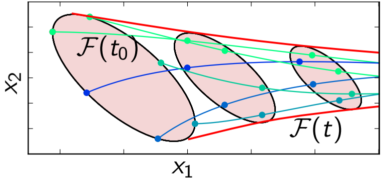

How to build funnels?

- Needs specific competences...

* Linear Quadratic Regulator

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  - First easy example: trajectory of the form $e^{-t} \cdot \mathbf{x}_0$

$$\text{For every } W \subseteq \mathbb{R}^d \text{ with } \mathbf{x}_0 \in W,\ \mathcal{F}_W : t \mapsto \{e^{-t} \cdot \mathbf{w} \mid \mathbf{w} \in W\}$$

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  - More generally, LQR* funnels $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$, with $\mathbf{x} : \mathbb{R} \to \mathbb{R}^d$

$$\mathcal{F}_\alpha : t \mapsto \{\mathbf{x}_{\mathrm{ref}}(t) + \mathbf{x}_\Delta \mid V(\mathbf{x}_\Delta) \leq \alpha\}$$

with V a Lyapunov function

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    with V a Lyapunov function
    - $\mathcal{F}_\alpha$ is a fixed d-dimensional ellipsoid centered on the reference trajectory
    - Those enjoy absorption properties:

      $$V(\mathbf{x}_\Delta(t + \delta t)) \leq e^{-\beta \cdot \delta t} V(\mathbf{x}_\Delta(t))$$

$$\mathcal{F}_{\alpha_1} \overset{\delta_1}{\leadsto} \mathcal{F}_{\alpha_2} \overset{\delta_2}{\leadsto} \mathcal{F}_{\alpha_3} \overset{\delta_3}{\leadsto} \dots \quad \text{with } \alpha_1 > \alpha_2 > \alpha_3 > \dots$$

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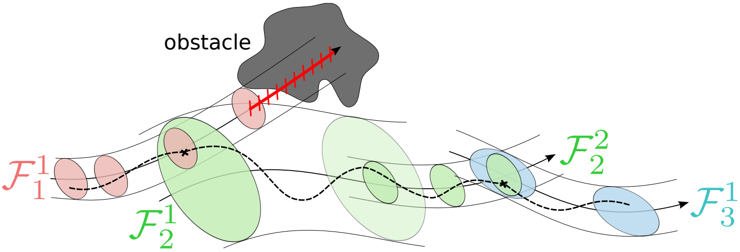
Example



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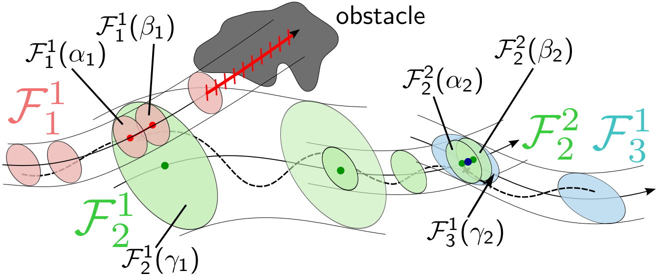
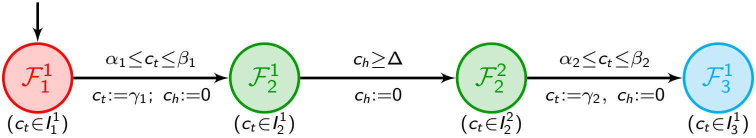
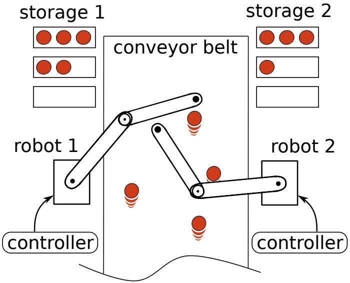


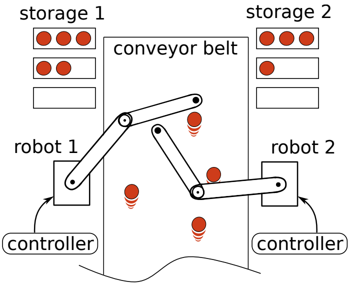
c_t: positional clock; c_h: local clock



Summary

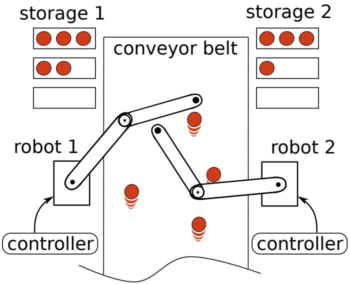

storage 1
storage 2
conveyor belt
robot 1
robot 2
controller
controller

Summary



$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

Summary



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$\leftarrow$ winning (optimal) strategy

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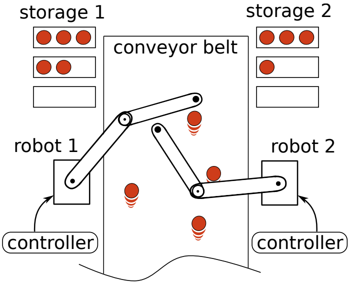
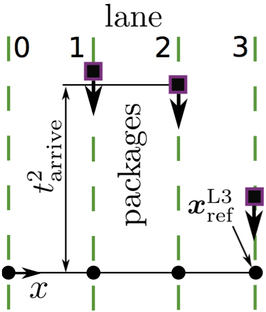


$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

safe (good) controller $\leftarrow$ winning (optimal) strategy

A pick-and-place example

1d point mass



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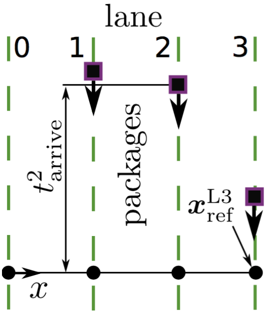
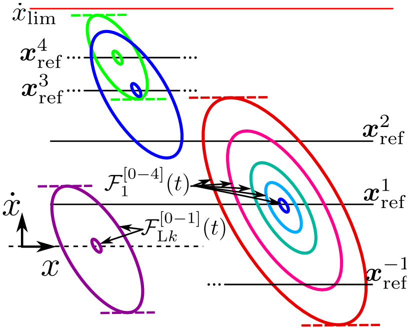
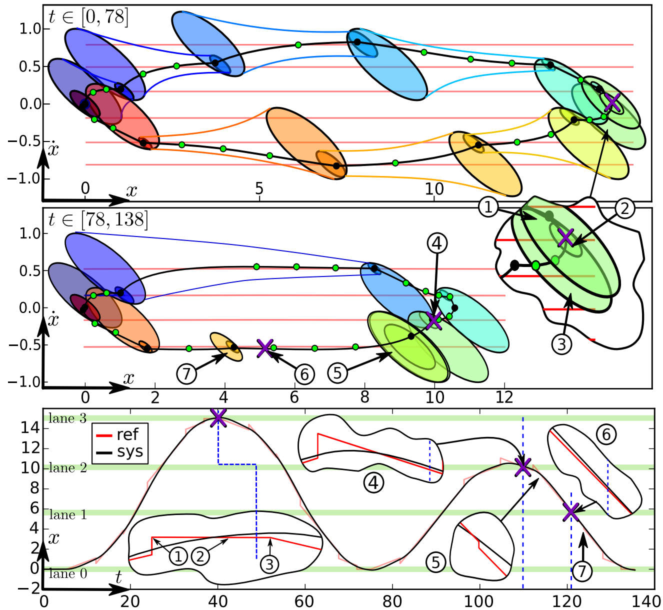


Funnel system




$t \in [0, 78]$
$t \in [78, 138]$
$\dot{x}$
x
t
lane 3
lane 2
lane 1
lane 0
ref
sys
1
2
3
4
5
6
7

Current challenges

For control people

- Handle more non-linear systems (automatically build control funnels)

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For control people

- Handle more non-linear systems (automatically build control funnels)

For us

- Does not scale up very well so far (huge timed automata models)
    - Build the model on-demand?
      But, can we give guarantees (optimality) when only part of the model has been built?
    - Develop specific algorithms for the special timed automata we construct?
    - Note: Reachability is indeed in NLOGSPACE...
- Implement efficient approx. algorithm for weighted timed games

Outline

Conclusion

Summary of the talk

- Basics of timed automata verification
- Relevant extensions for applications: weights, games, mix of both
  - We looked at decidability and limits
  - We mentioned algorithmics and tools
- Timed automata can be used as abstractions for more complex systems

Conclusion

Current challenges

- Various theoretical issues
  - Decidability and approximability of weighted timed automata and games
  - New approaches (tree automata, reachability relations) might give a new light on the verification of timed systems
  - Robustness and implementability
- Continue working on algorithms, tools and benchmarks

Within ANR project Ticktac

  - Implementation of (weighted) timed games (good data structures, abstractions, etc.)
  - More applications with specific challenges (e.g. robotic problems)

ANR Ticktac webpage: `http://www.irisa.fr/sumo/ticktac/`