ACI «Sécurité Informatique» CORTOS

- CORTOS = Control and Observation of Real-Time Open Systems
- Participants: LSV + VERIMAG + IRCCyN
- Web: `http://www.lsv.ens-cachan.fr/aci-cortos/`

Thèmes du projet

- Algorithmes de synthèse de contrôleur
- Observation et détection de fautes
- Logiques pour exprimer le contrôle
- Contrôle optimal

Session Invitée

1. Introduction au contrôle des systèmes temps-réel
2. Observation partielle des systèmes temporisés
3. Implémentabilité des automates temporisés

Control of Timed Systems

K. Altisen[1], P. Bouyer[2], T. Cachat[3], F. Cassez[4], G. Gardey[4]

[1]VERIMAG Grenoble
[2]LSV Cachan
[3]LIAFA Paris
[4]IRCCyN Nantes

MSR'05

October 2005, Autrans, France

Outline of the talk

- Verification & Control
- Control of Finite Automata
- Timed Game Automata
- Symbolic Algorithms for Timed Game Automata
- Conclusion

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Verification and Control

Verification and Control

Does the system meet the specification ?

Verification and Control

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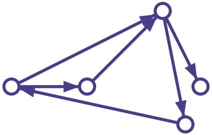
Modelling



$\square$ (not bad)

$S \models \phi$

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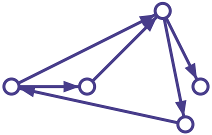

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Model Checking Problem

Does the closed system S satisfy ϕ ?

Verification and Control

Can we enforce the system to meet the specification ?

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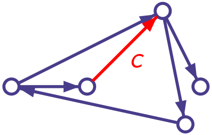

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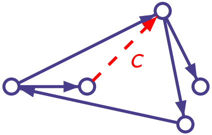

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Control Problem

Can the open system S be restricted to satisfy ϕ ?

Is there a Controller C s.t. $(S \parallel C) \models \phi$?

Verification and Control

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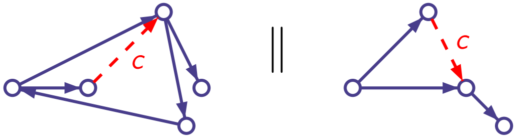

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$S \quad \| \quad C \quad \models \quad \phi$

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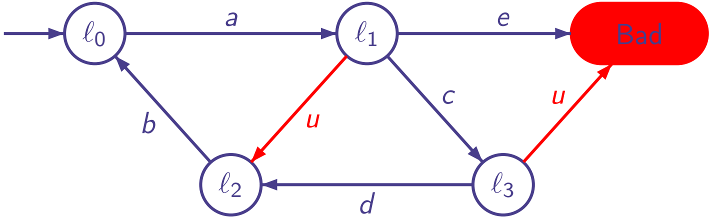
Control of Discrete Event Systems

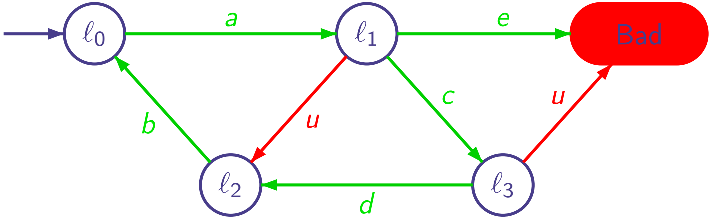


- Introduced by Ramadge & Wonham [Ramadge, 87]
- Discrete Event System = Finite Automaton with

> Controllable (Act_c) and Uncontrollable (Act_u) actions

- Example of Control Objective: "avoid state Bad"
- Means: disable some controllable transitions at the right time
  Ramadge & Wonham Theory is based on Language Theory [Ramadge, 89, Thistle, 94]

Control and Game



Open System = 2-player game, Controller (C) vs Environment (E)

- Controller does Act_c moves, Environment does Act_u moves
- Control Objective = Winning condition on the game

  "Avoid bad states" (safety) or "Enforce good states" (reachability)
- Control Problem: find a strategy for the controller to win the game
- Various types of game models for C and E
  - Finite or pushdown or counter automata ...
  - Timed or hybrid automata

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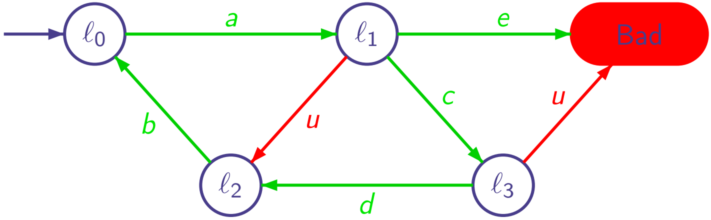

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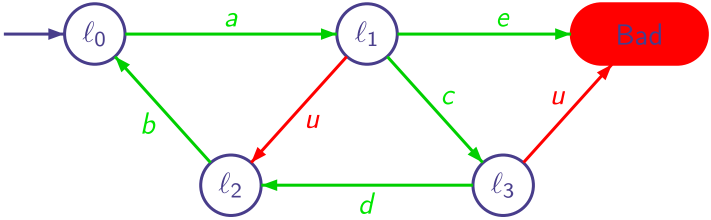

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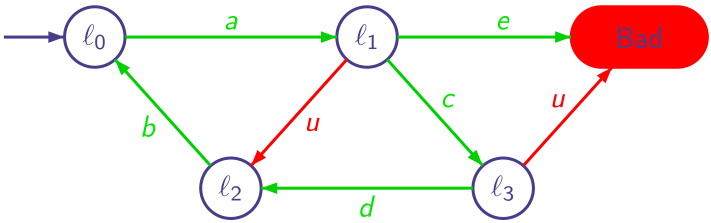

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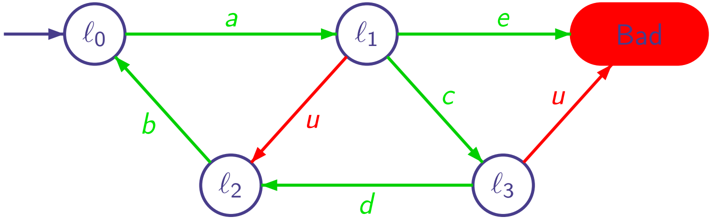

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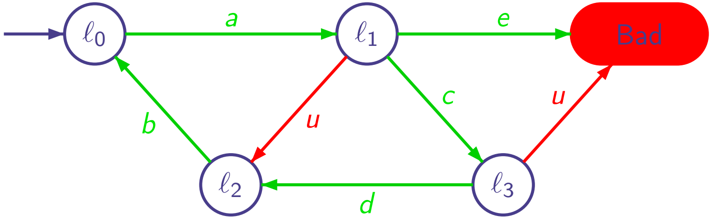

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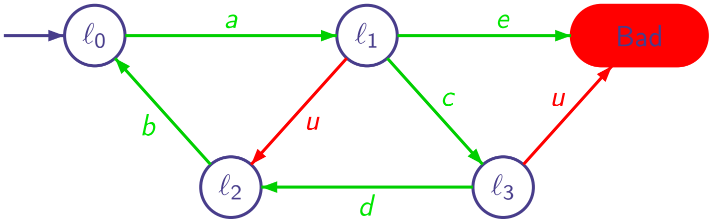

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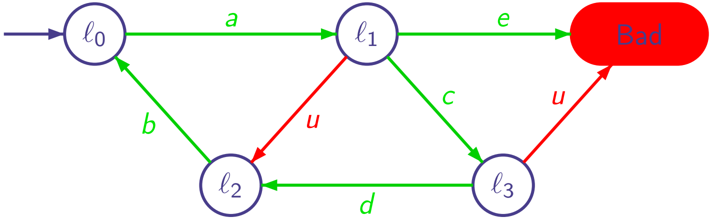

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Problems of Interest

Verification Problem (or Model Checking Problem)

Input: a model of the closed system S and a property φ
Problem: Does S satisfy φ ?

Control Problem (CP)

Input: a model of the open system (game) G and a property φ
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Control Synthesis Problem (CSP)

Input: a model of the open system (game) G and a property φ
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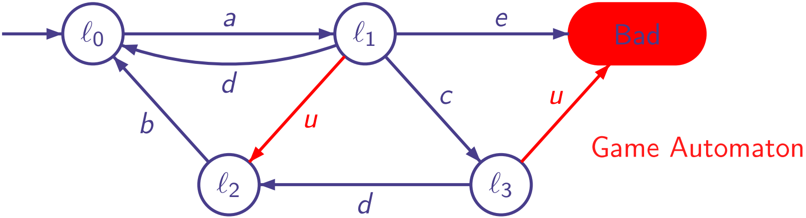
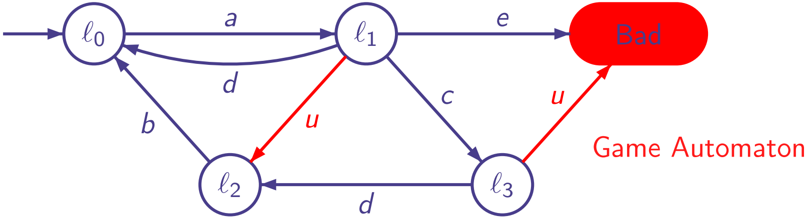
Game Automata, Strategies & Winning States



Strategy

- A strategy f gives for each finite run the controllable action to take. We assume full observability of the system

Game Automata, Strategies & Winning States



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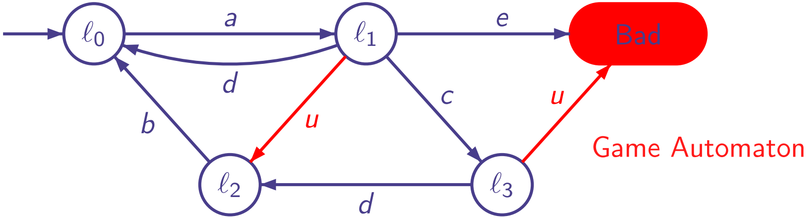

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Example of Strategies:

$$f(\ell_0) = a$$
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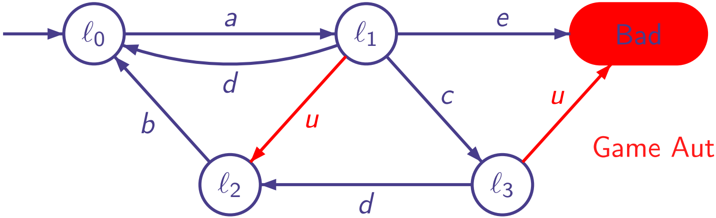
Game Automata, Strategies & Winning States



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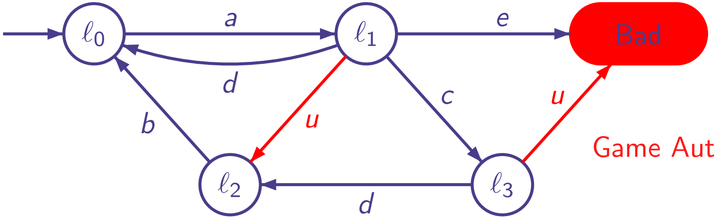

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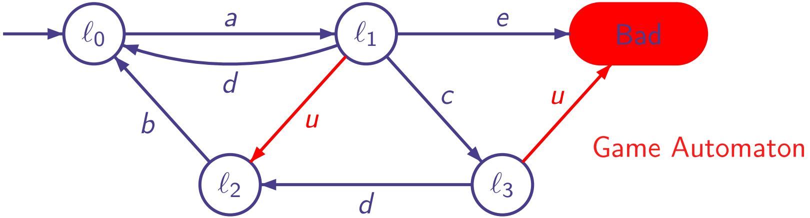

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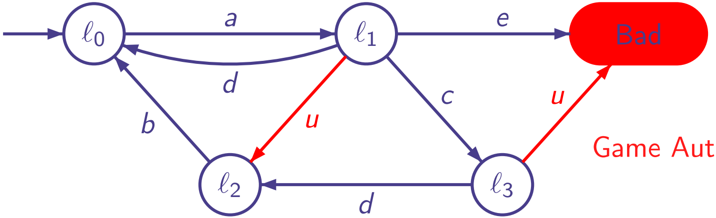

Game Automaton

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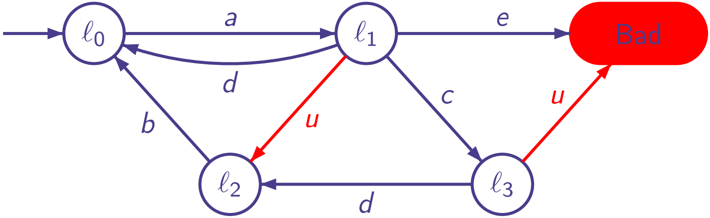
Winning States

A state s is winning if there exists a winning strategy from s.

Controllable Predecessors



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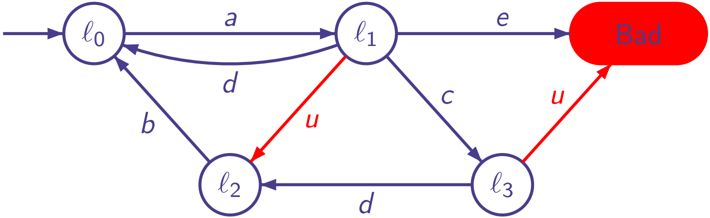
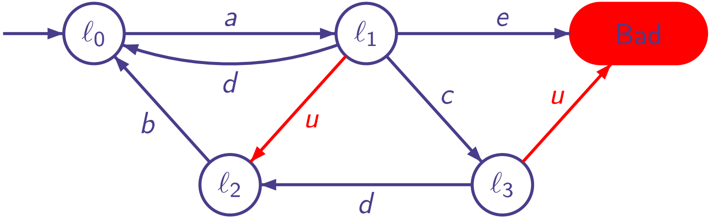


$\pi(X)$ = states from which one can enforce X with a controllable action

$$\pi(X) = \mathrm{Pred}^{\mathrm{Act}_c}(X) \setminus \mathrm{Pred}^{\mathrm{Act}_u}(\overline{X})$$

Controllable Predecessors



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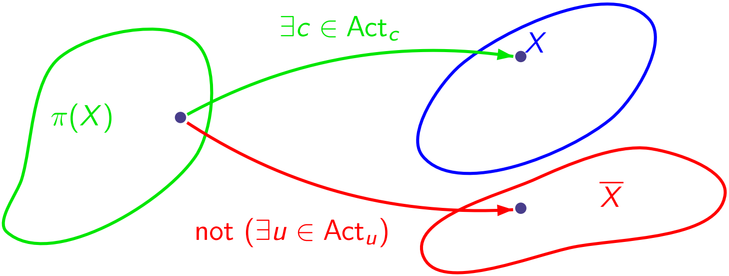

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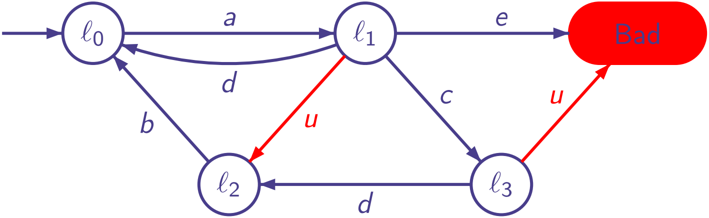

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Some Values of the π Operator

- $\pi(\{\ell_3\}) = \emptyset$
- $\pi(\{\ell_1\}) = \{\ell_0\}$
- $\pi(\{\ell_0, \ell_1\}) = \{\ell_0, \ell_2\}$
- $\pi(\{\ell_0, \ell_1, \ell_2\}) = \{\ell_0, \ell_1, \ell_2\}$

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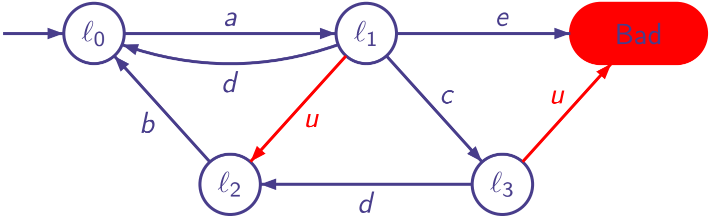

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A Fixpoint Characterization of Winning States:

1. let φ be a set of safe (good) states and G a game
2. let W^* be the greatest fixpoint of $h(X) = \varphi \cap \pi(X)$
3. W^* is the set of winning states for (G, φ)

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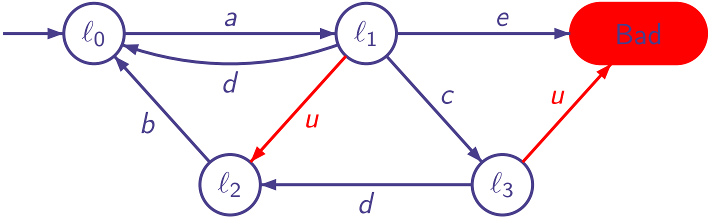

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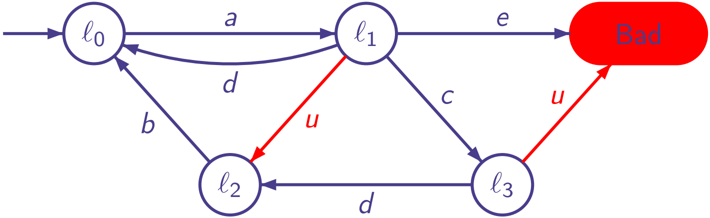

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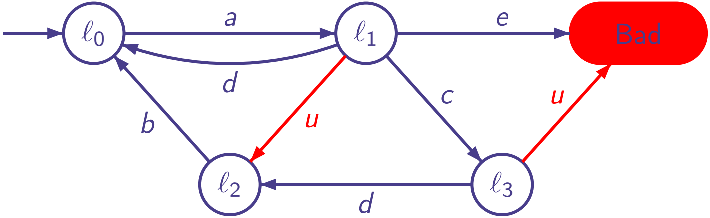

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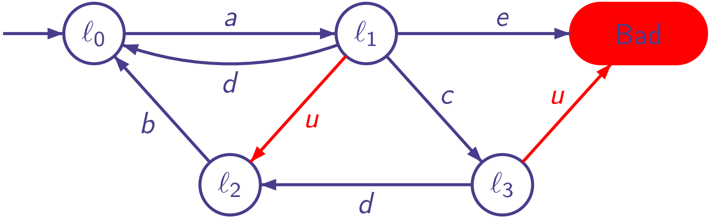

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- CP: check that $\ell_0 \in W^*$

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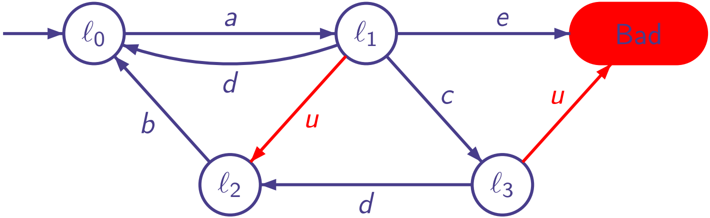

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- CP: check that $\ell_0 \in W^*$
- CSP: Given W^* and G, we can build a winning strategy

Results for Finite Games

Given G a finite game, φ a control objective

Theorem (Positional Strategies are Sufficient)

Positional (or memoryless) strategies suffice to win ω-regular games.
The number of states of C is $\leq$ number of states of G.

Results for Finite Games

Given G a finite game, φ a control objective

The fixpoint computation of W^* terminates

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Theorem (Effectiveness of CSP)

Strategy synthesis is effective. We can build a finite automaton (controller) C that specifies a winning strategy.

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Results for Finite Games

Given G a finite game, φ a control objective

Theorem (CP is Decidable)

CP is decidable for ω-regular objectives.

Theorem (Effectiveness of CSP)

Strategy synthesis is effective. We can build a finite automaton (controller) C that specifies a winning strategy.

Theorem (Positional Strategies are Sufficient)

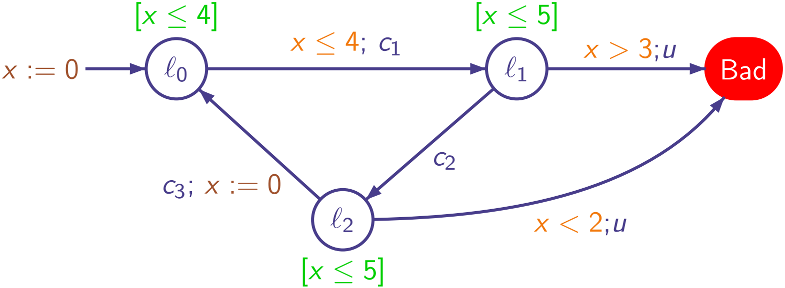
Positional (or memoryless) strategies suffice to win ω-regular games. The number of states of C is $\leq$ number of states of G.

Add Dense Time ... CP and CSP ?

Outline

- Verification & Control
- Control of Finite Automata
- **Timed Game Automata**
- Symbolic Algorithms for Timed Game Automata
- Conclusion

Timed Automata [Alur & Dill'94]



Runs = sequence of discrete and time steps

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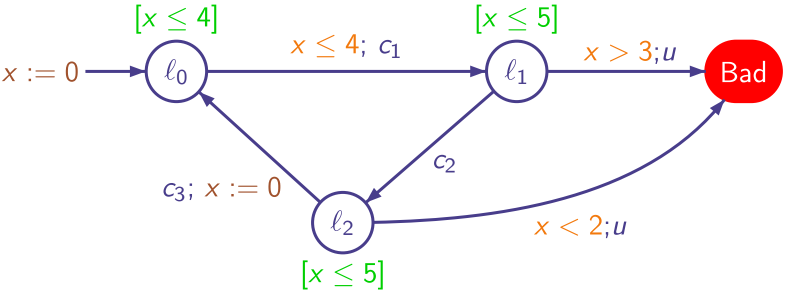

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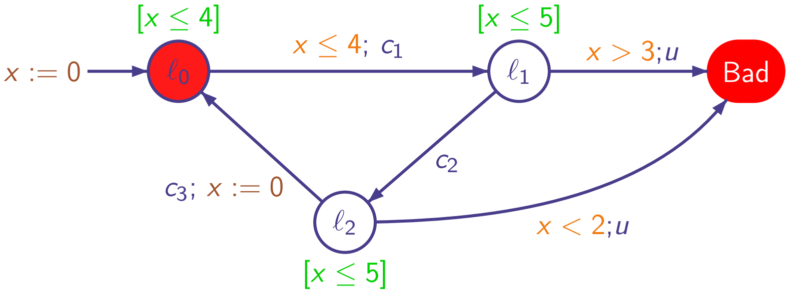

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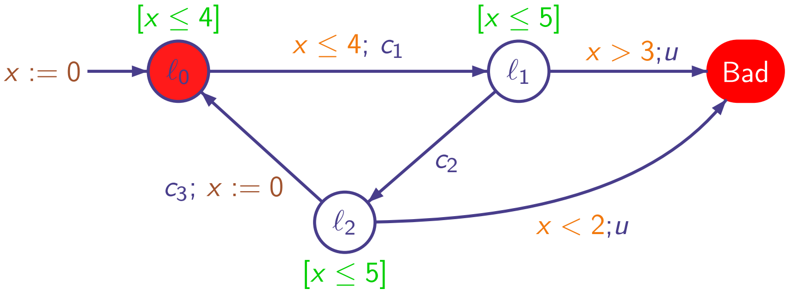

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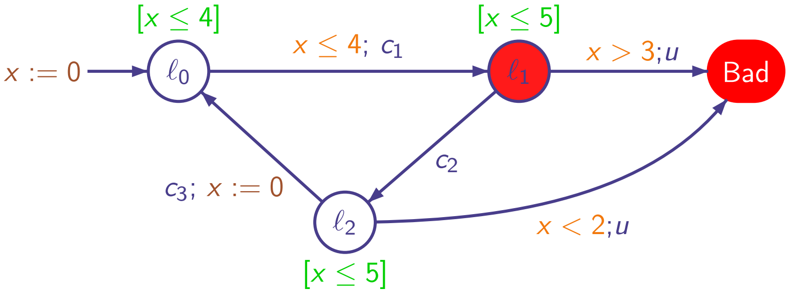

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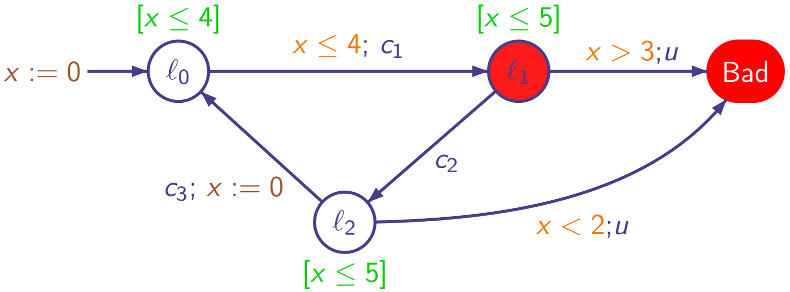

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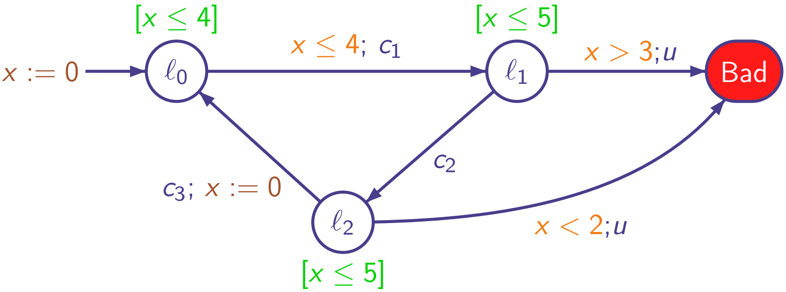

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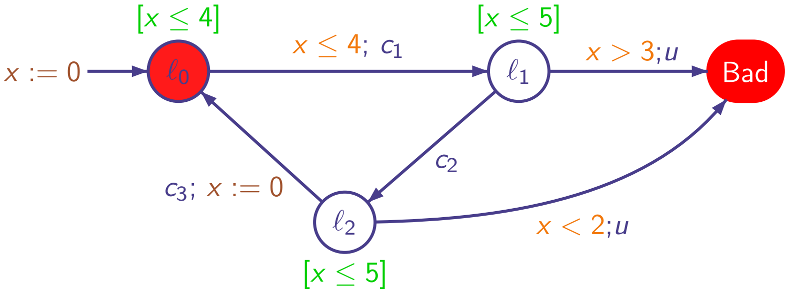

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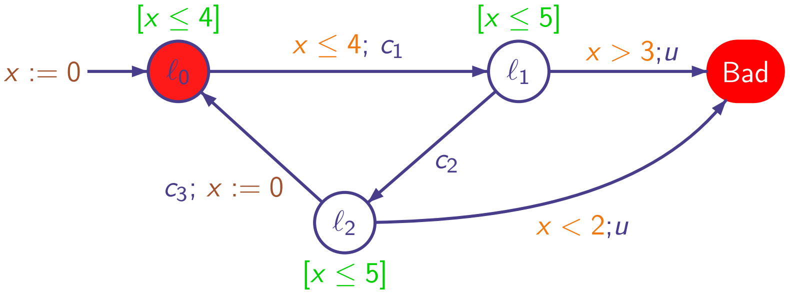

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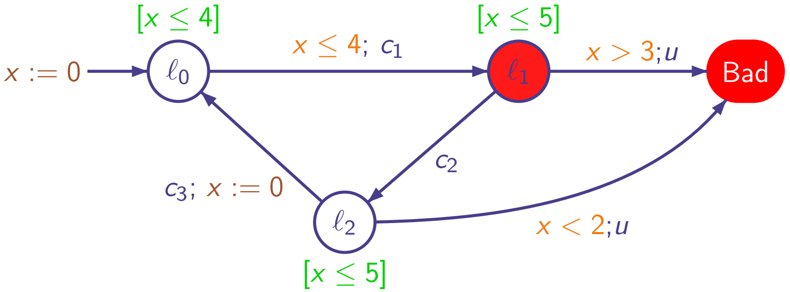

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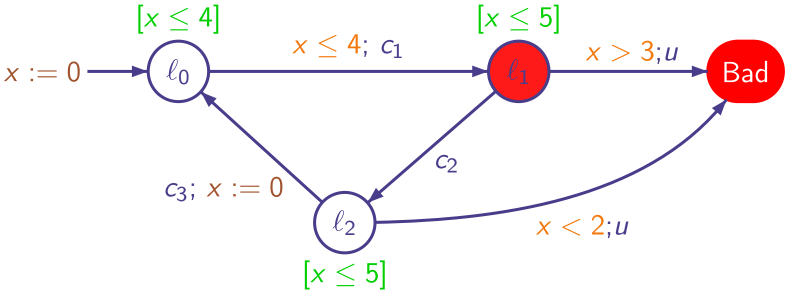

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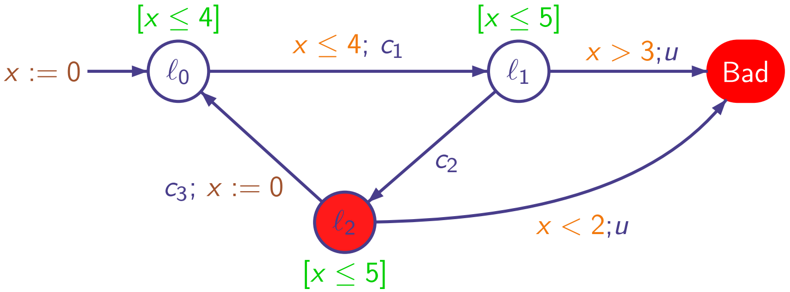

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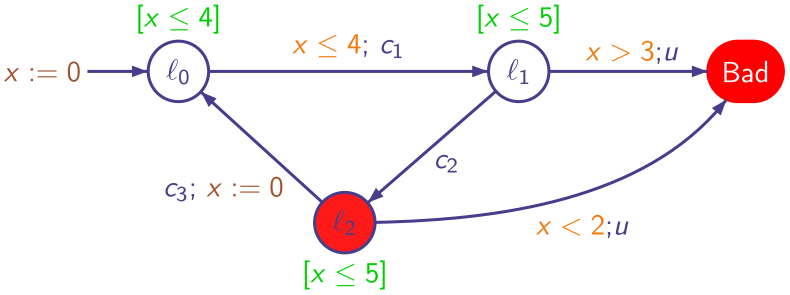

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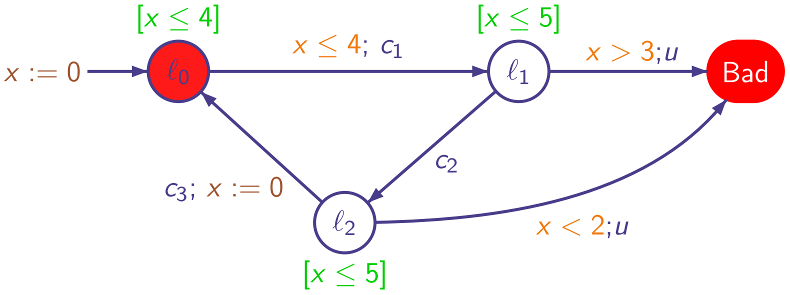

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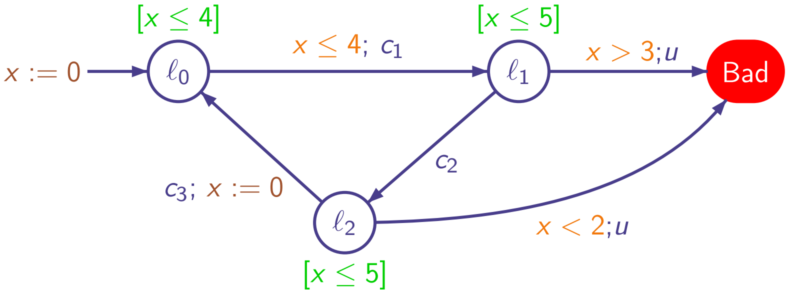

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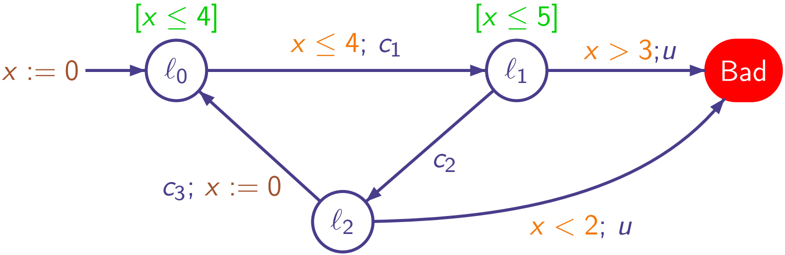

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- The controller continuously observes the system
  time elapsing and discrete moves are observable
- It has the choice between two types of moves:
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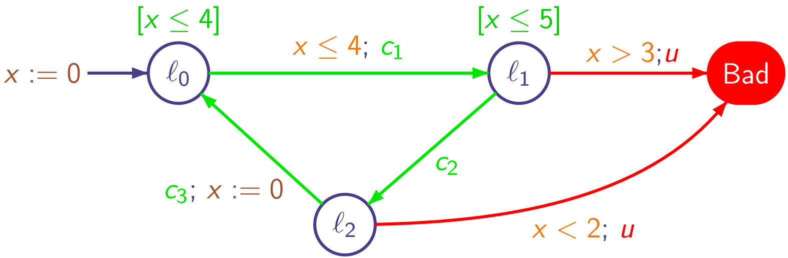

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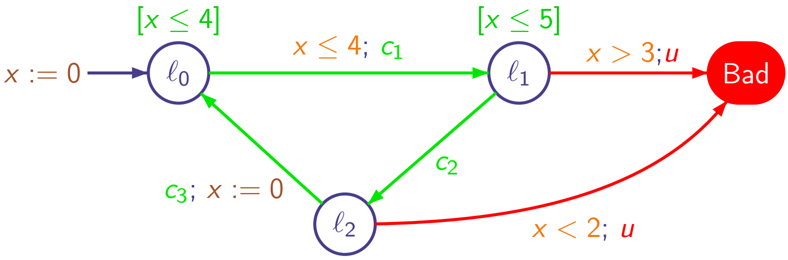

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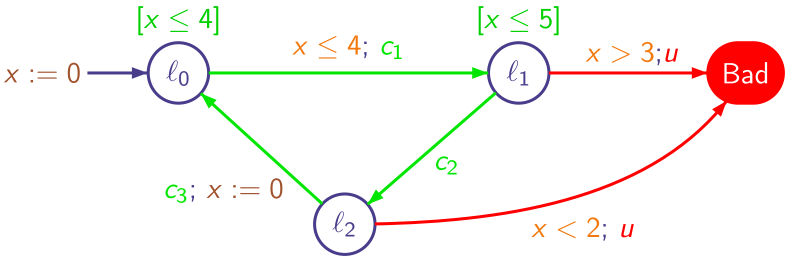

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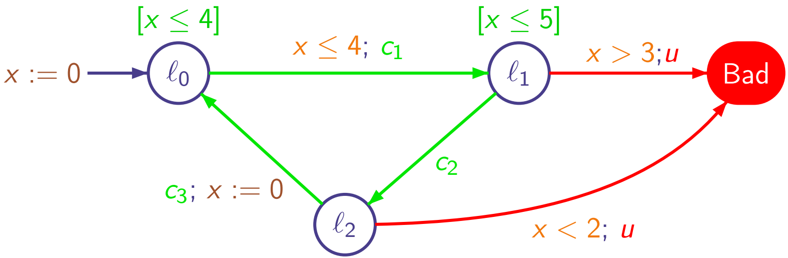

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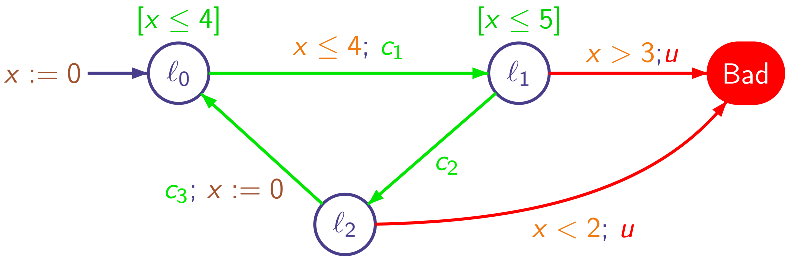

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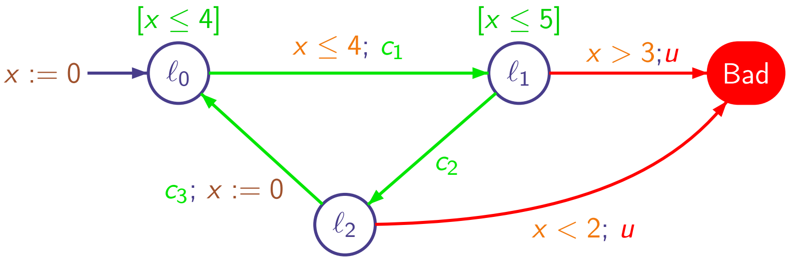

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How to Deal with Dense-Time ?

- Infinite state systems

Symbolic representation of states

- A strategy (or controller) can choose to wait

Add a special wait action

- Dense time $\cdots$ the controller can be unfair
  - block time
  - do infinitely many actions in a bounded time
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Implementation Issues

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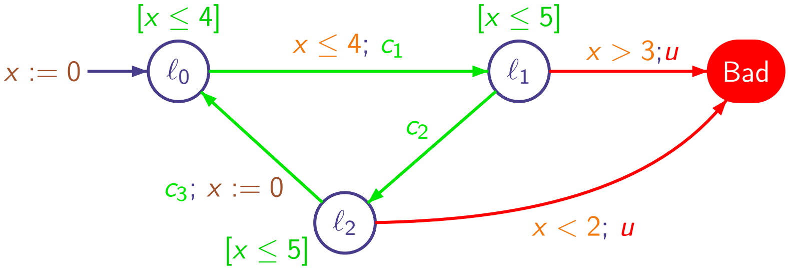
Strategies and Winning States



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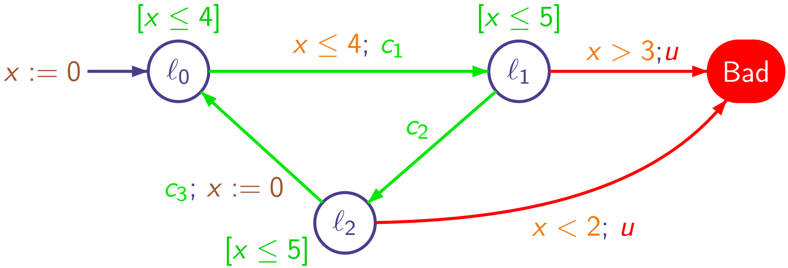

The strategy f: "Always wait as long as the system permits"

ρ_1 : $(\ell_0, 0) \xrightarrow{4} (\ell_0, 4) \xrightarrow{c_1} (\ell_1, 4) \xrightarrow{0.5} (\ell_1, 4.5) \xrightarrow{u} (\text{Bad}, 4.5)$

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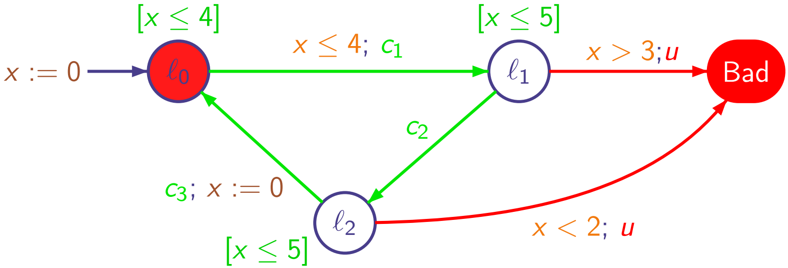

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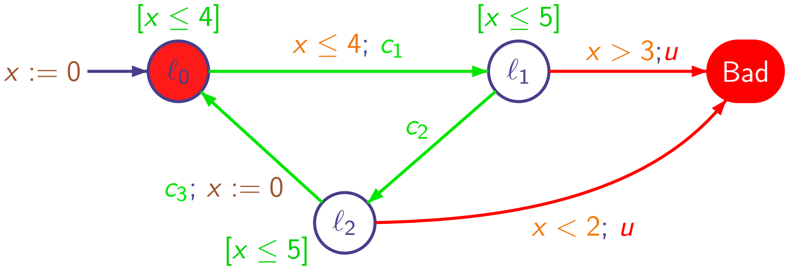

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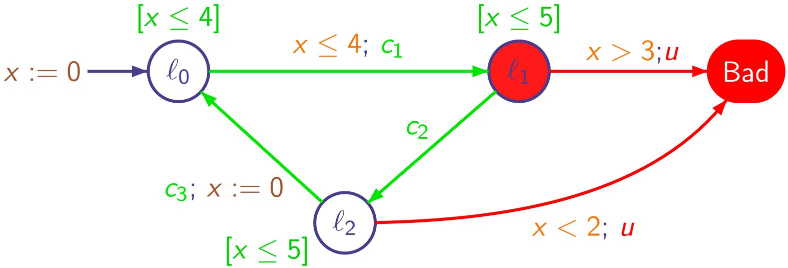
ρ_1 : $(\ell_0, 0) \xrightarrow{4} (\ell_0, 4) \xrightarrow{c_1} (\ell_1, 4) \xrightarrow{0.5} (\ell_1, 4.5) \xrightarrow{u} (\text{Bad}, 4.5)$

ρ_2 : $(\ell_0, 0) \xrightarrow{4} (\ell_0, 4) \xrightarrow{c_1} (\ell_1, 4) \xrightarrow{1.0} (\ell_1, 5) \xrightarrow{c_2} (\ell_2, 5) \xrightarrow{c_3} (\ell_0, 0) \cdots$

Strategies and Winning States



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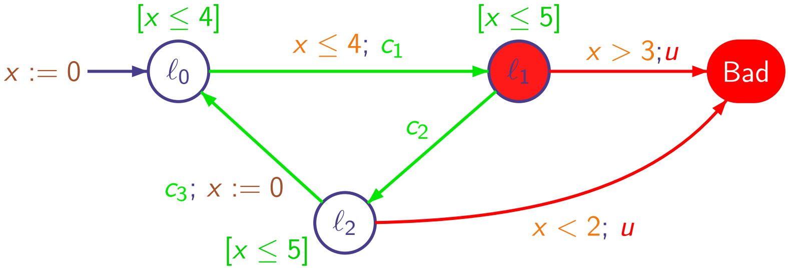
Strategies and Winning States



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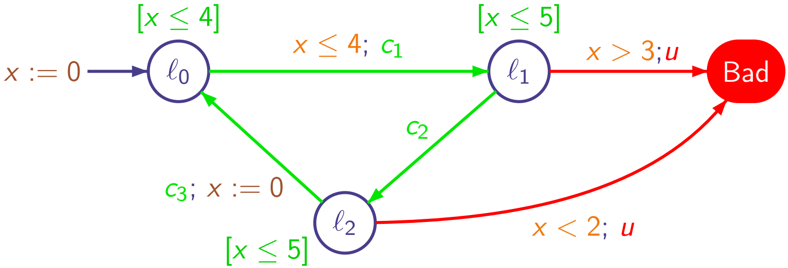
Strategies and Winning States



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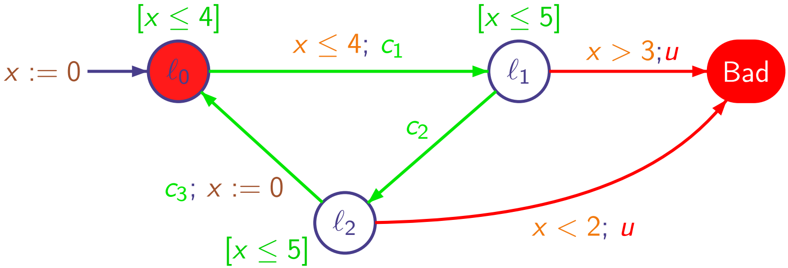
Strategies and Winning States



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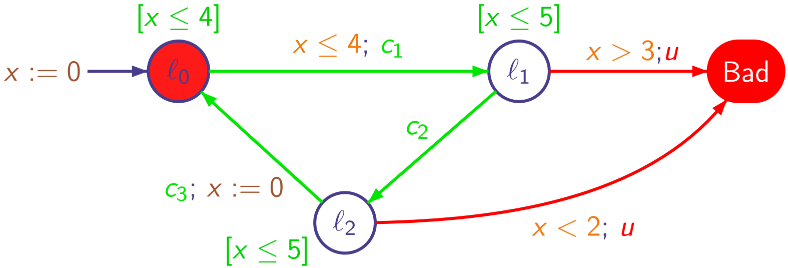
Strategies and Winning States



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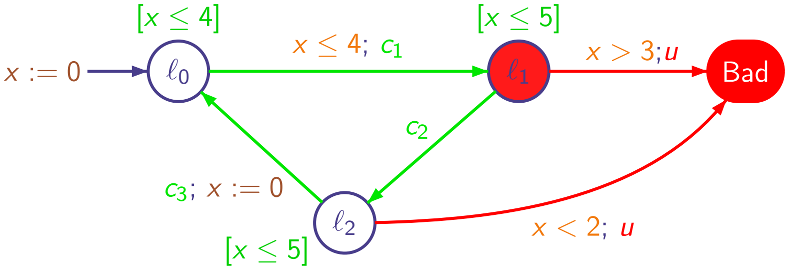
Strategies and Winning States



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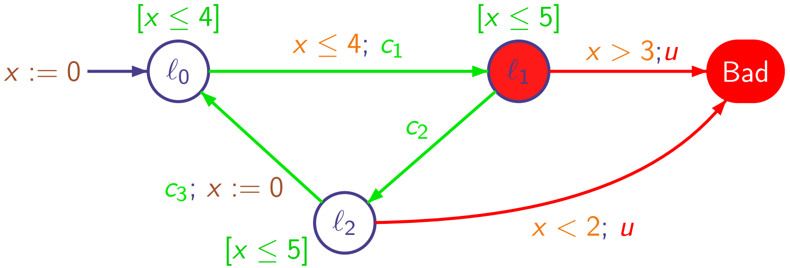
Strategies and Winning States



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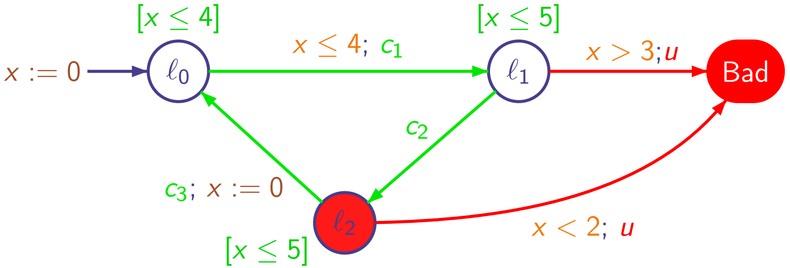
Strategies and Winning States



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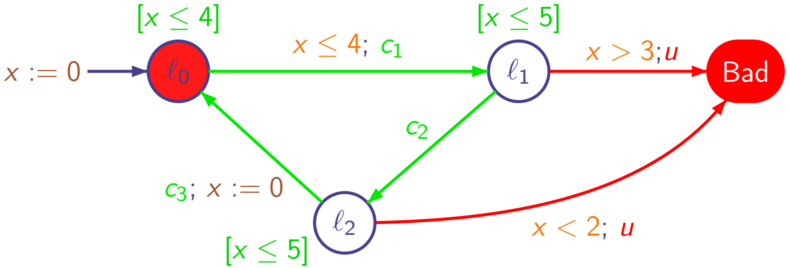
Strategies and Winning States



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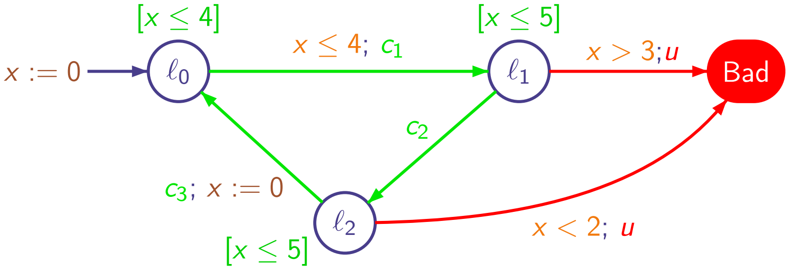
Strategies and Winning States



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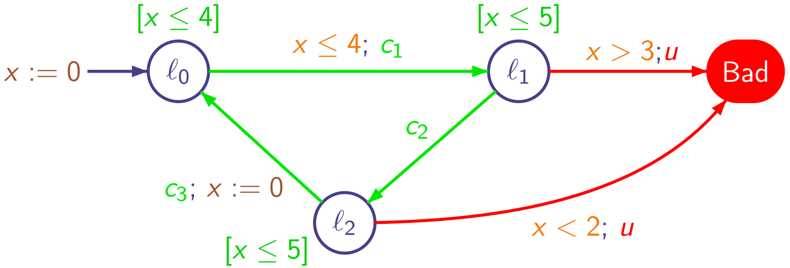
Strategies and Winning States



A winning strategy f'

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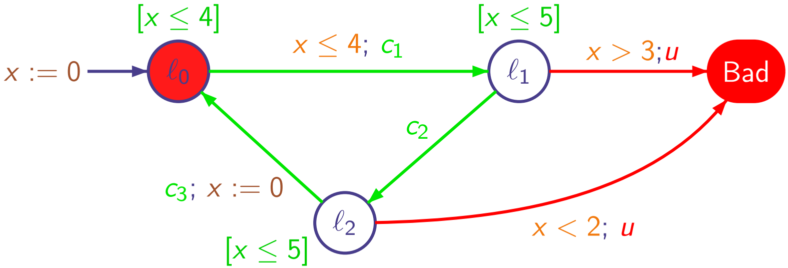
$\rho: \quad (\ell_0, 0) \xrightarrow{2} (\ell_0, 2) \xrightarrow{c_1} (\ell_1, 2)$

Strategies and Winning States



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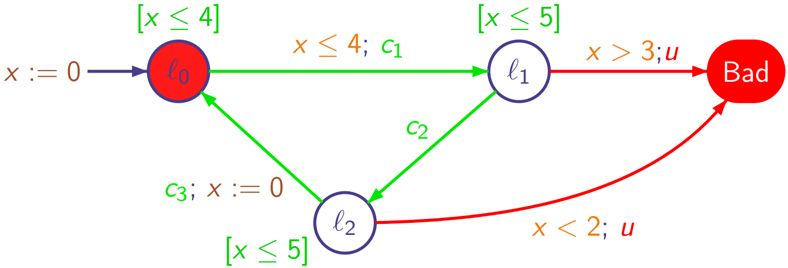
Strategies and Winning States



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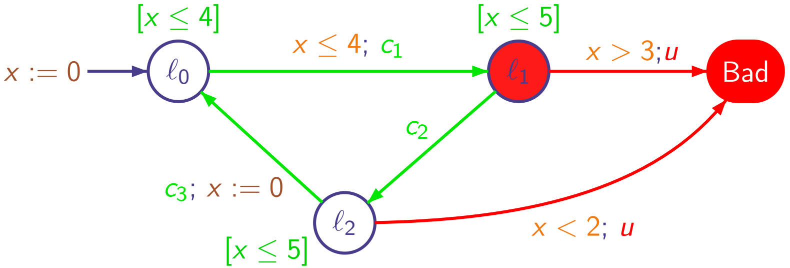

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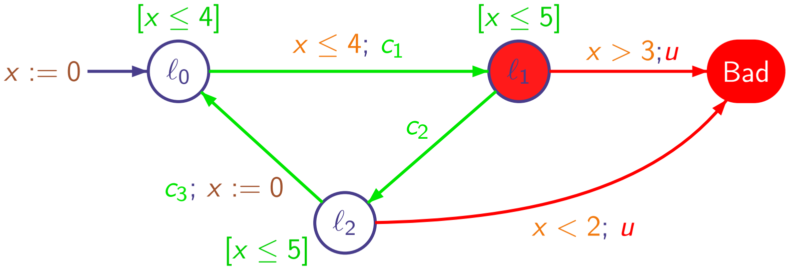
Strategies and Winning States



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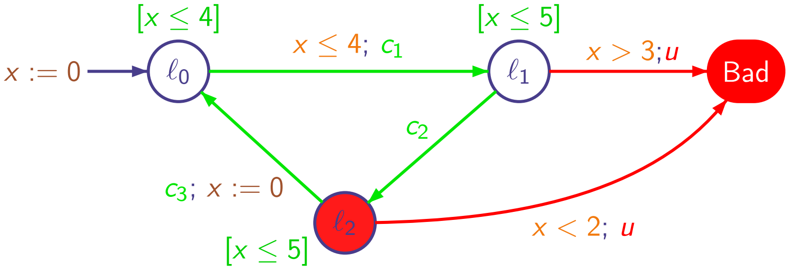
Strategies and Winning States



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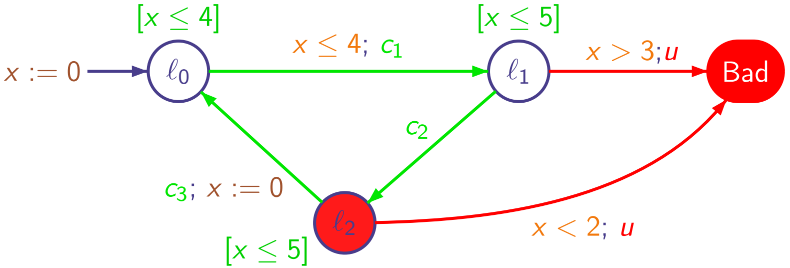
Strategies and Winning States



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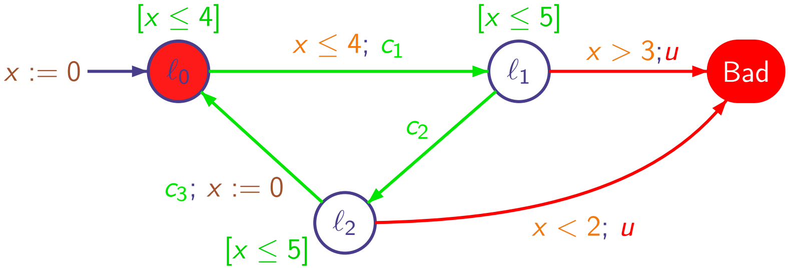
Strategies and Winning States



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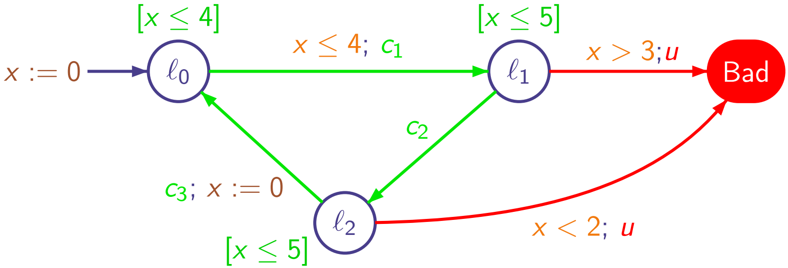
Strategies and Winning States



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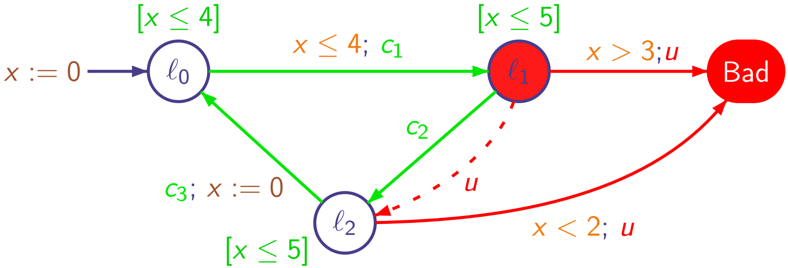
Strategies and Winning States



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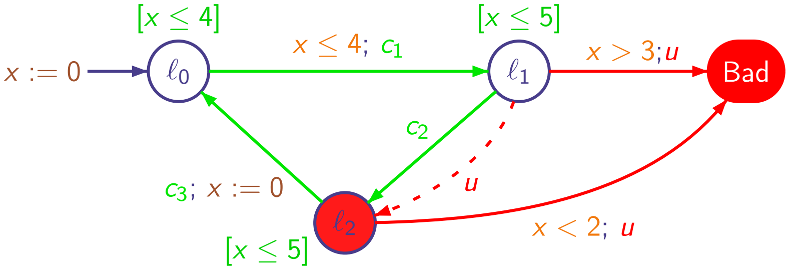
Strategies and Winning States



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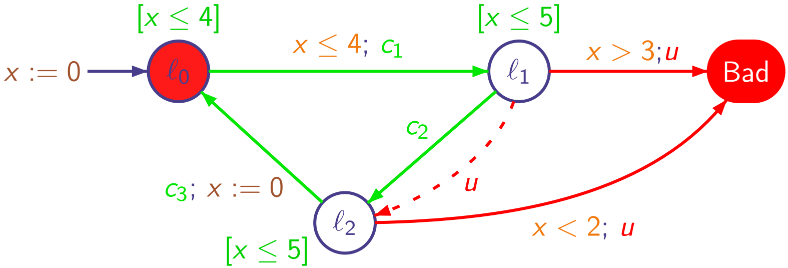
Strategies and Winning States



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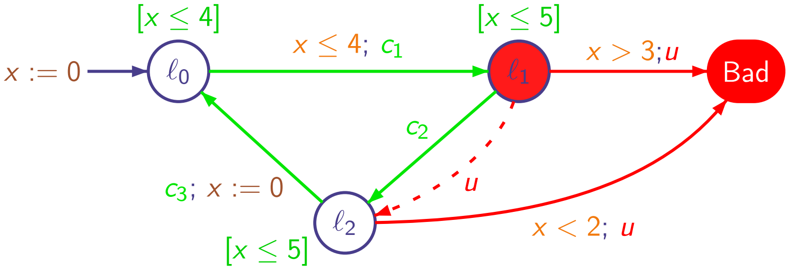
Strategies and Winning States



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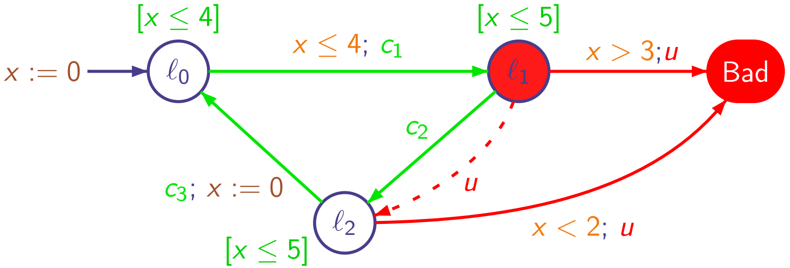
Strategies and Winning States



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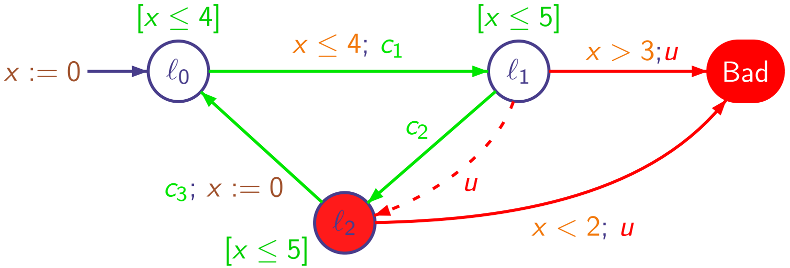
Strategies and Winning States



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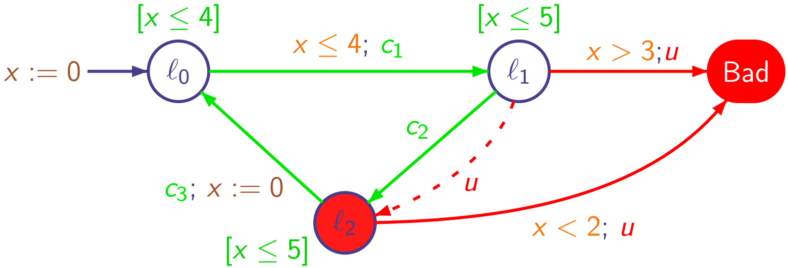
Strategies and Winning States



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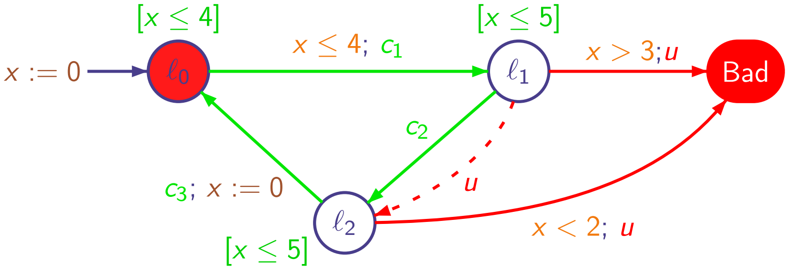
Strategies and Winning States



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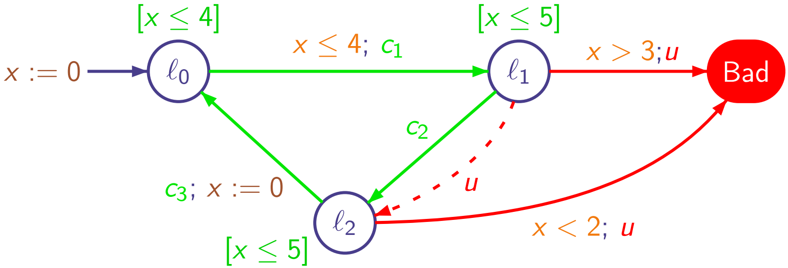
Strategies and Winning States



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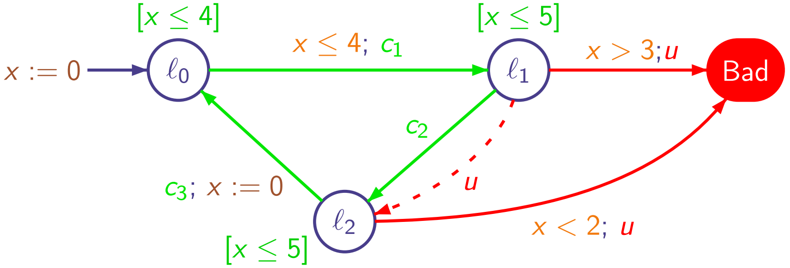
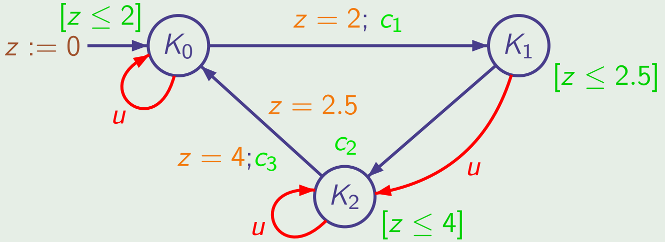
Strategies and Winning States



The Strategy f' as a Timed Automaton



Outline

- Verification & Control
- Control of Finite Automata
- Timed Game Automata
- **Symbolic Algorithms for Timed Game Automata**
- Conclusion

Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ a the set of states of the TGA $\qquad q = (\ell, v) \in Q$
- Discrete predecessors of $X \subseteq Q$ by an action a:
$$\mathrm{Pred}^a(X) = \{q \in Q \mid q \xrightarrow{a} q' \text{ and } q' \in X\}$$
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X is a state predicate

- $\mathrm{cPred}(X) = \bigcup_{c \in \mathrm{Act}_c} \mathrm{Pred}^c(X)$ $\qquad$ $\mathrm{uPred}(X) = \bigcup_{u \in \mathrm{Act}_u} \mathrm{Pred}^u(X)$
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- $\mathrm{Pred}_\delta(X, Y)$: Time controllable predecessors of X avoiding Y:

q $\qquad\qquad$ $q' \in X$

$\mathrm{Pred}_\delta(X, Y)$ is effectively computable for state predicates X, Y

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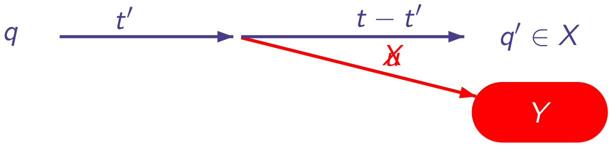

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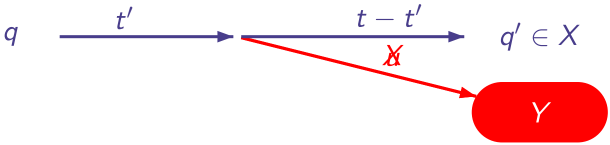

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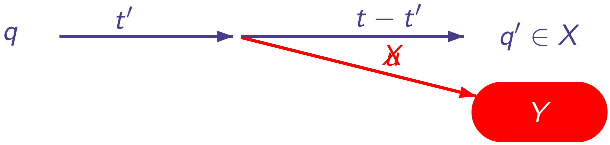

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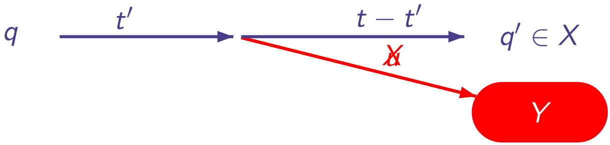

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Solving CP and CSP for Safety Timed Games

Symbolic Algorithm for Safety Timed Games

1. let φ be a State Predicate, G a timed game
2. let W^* be the greatest fixpoint of $h(X) = \varphi \cap \pi_\delta(X)$
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- CP: check that $(\ell_0, 0) \in W^*$
- CSP: by def. of π_δ there is a strategy

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The iterative computation of W^* terminates for (G, φ) with G a timed game automaton φ a ω-regular control objective.

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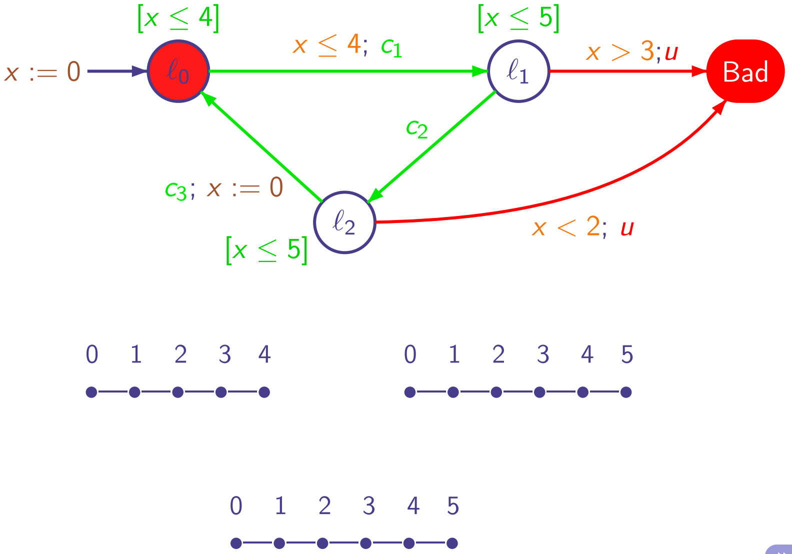
Theorem (Effectiveness of CSP)

If $(\ell_0, 0) \in W^*$ we can compute a positional winning strategy.

Example of Computation



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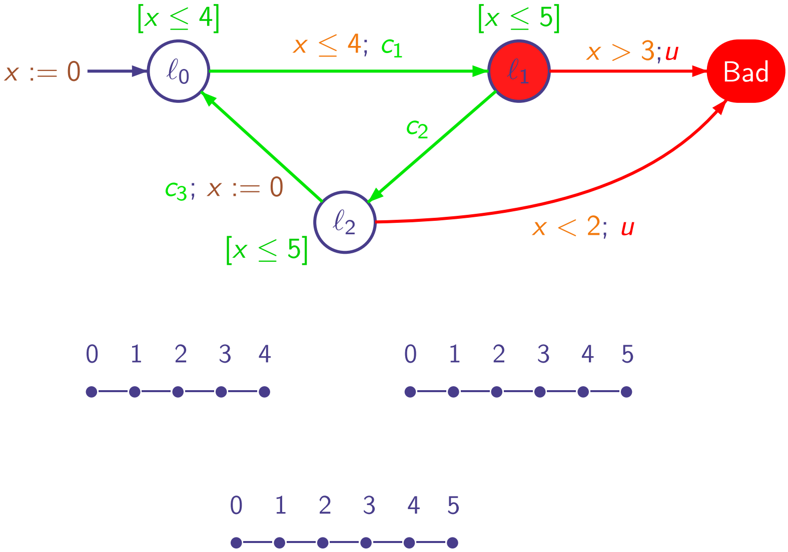

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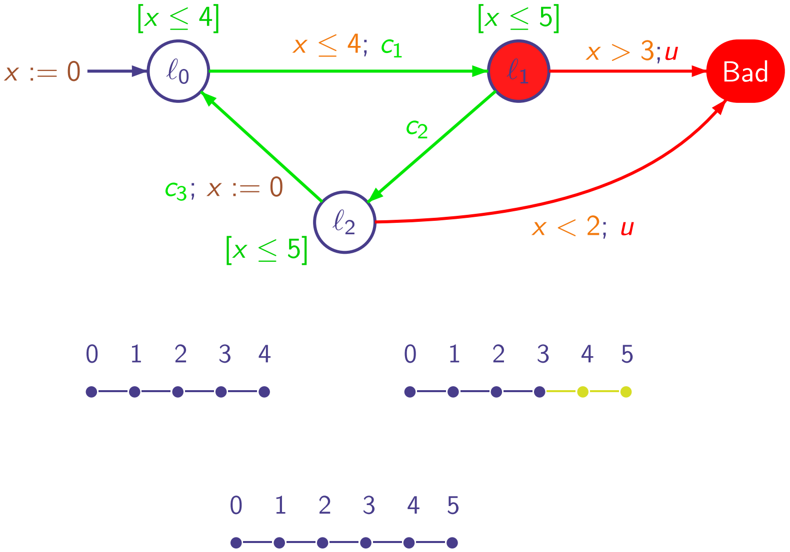

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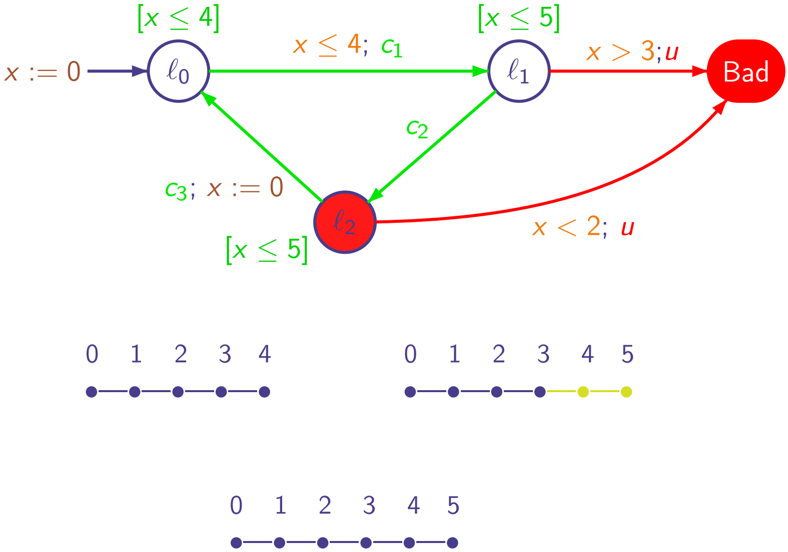

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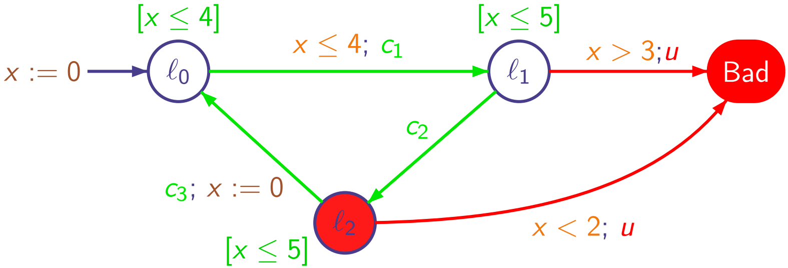

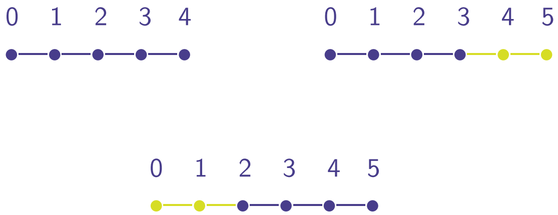

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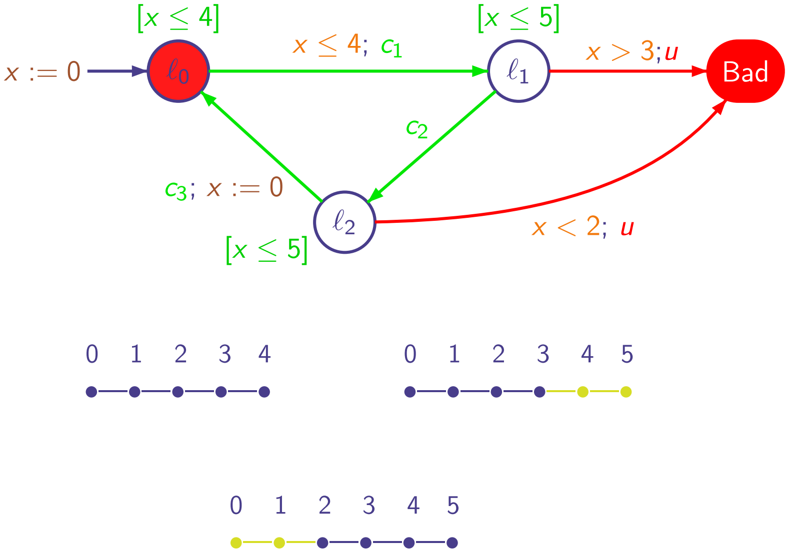

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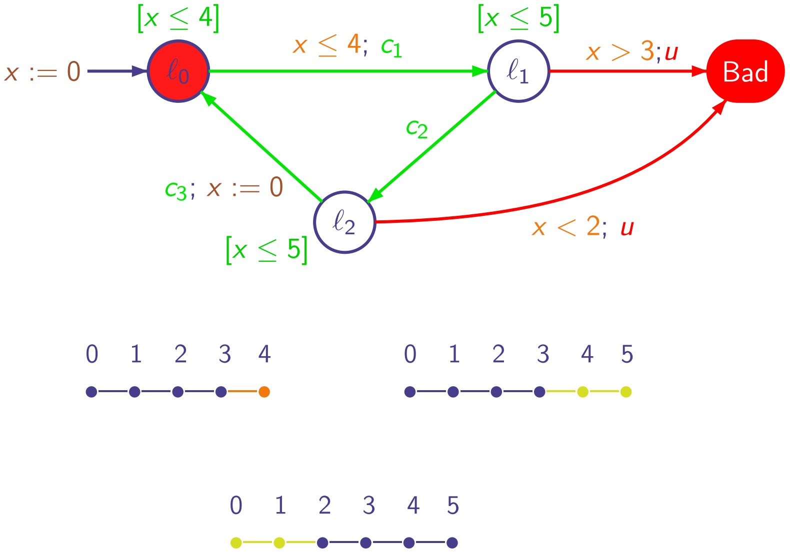

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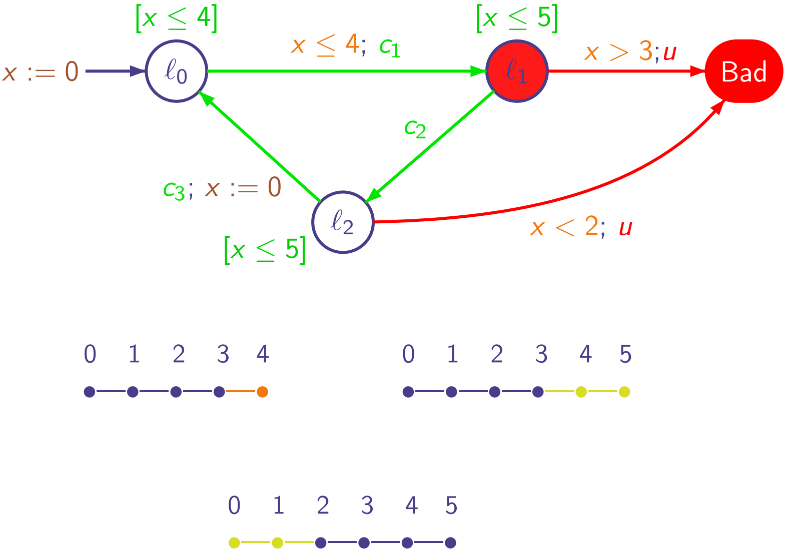

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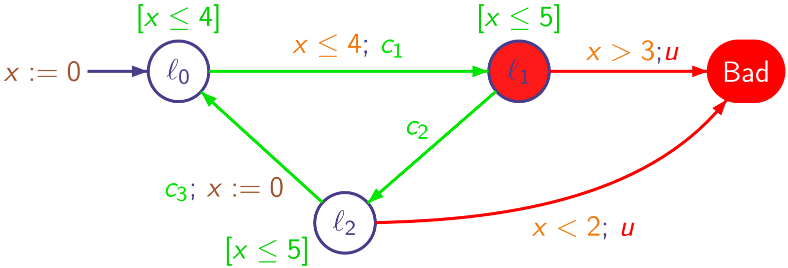

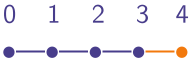

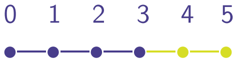

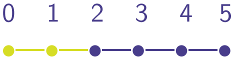

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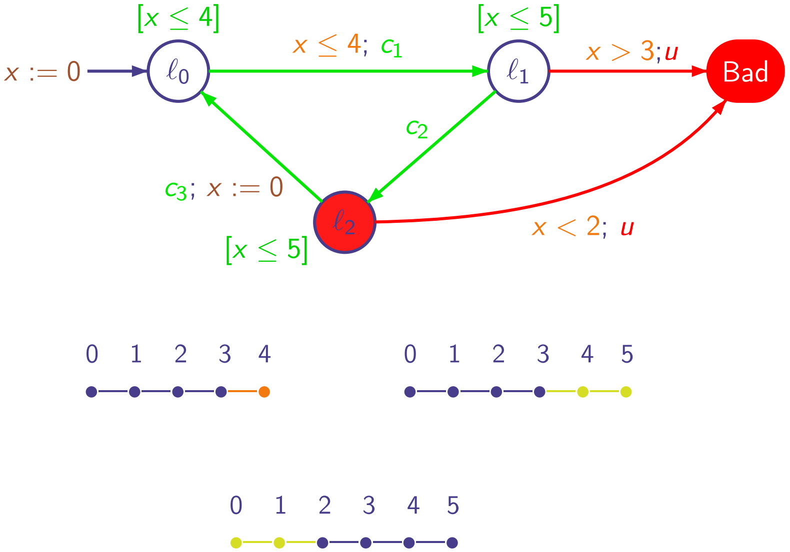

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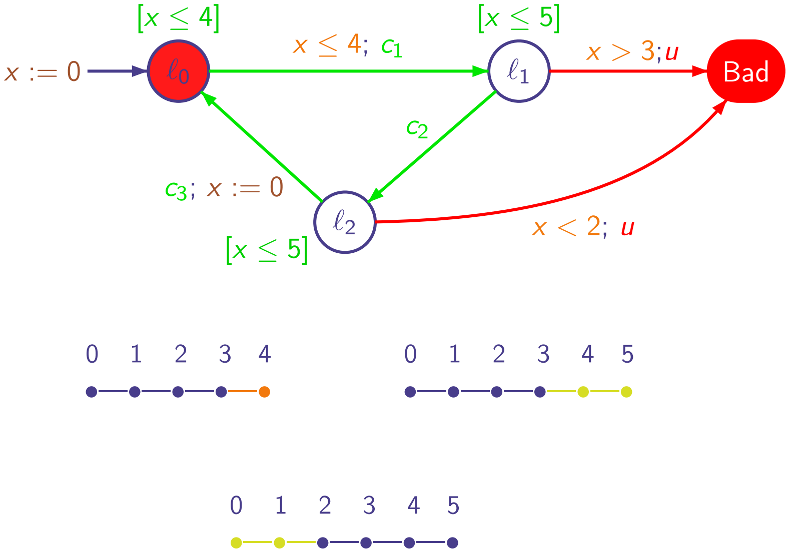

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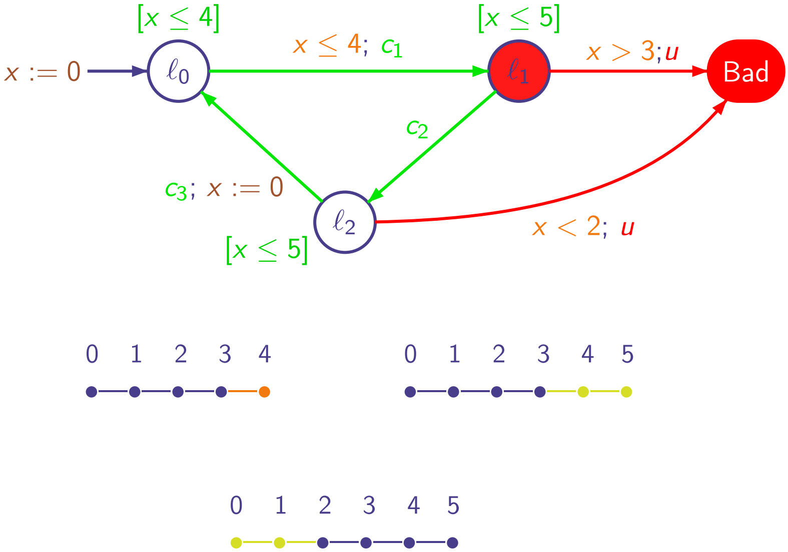

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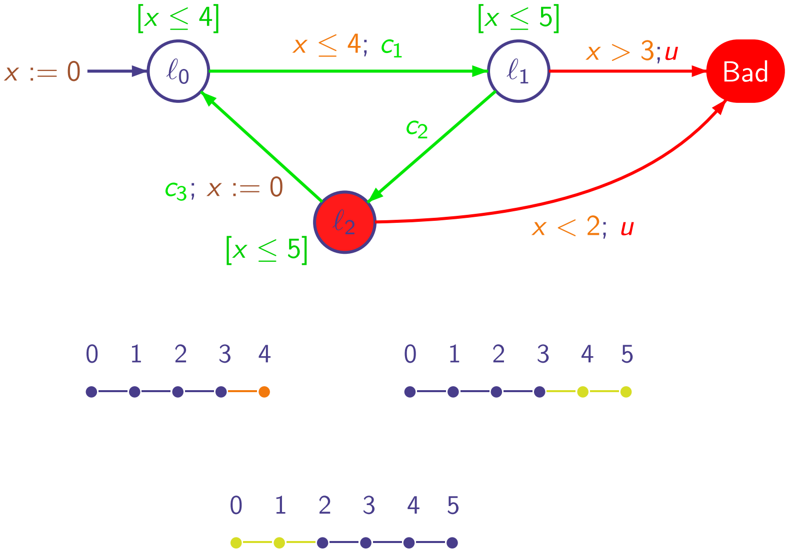

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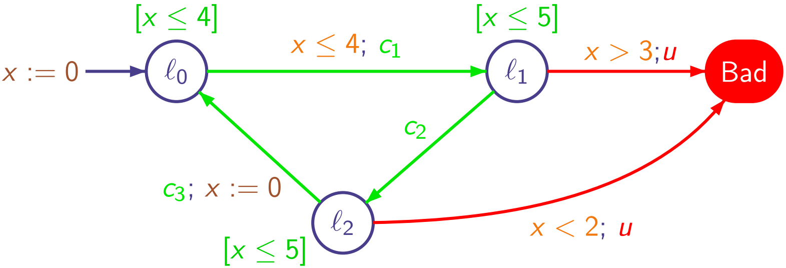

$z := 0$

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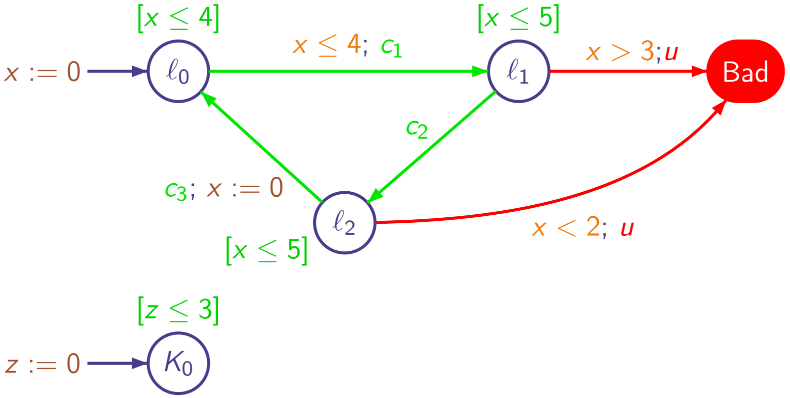

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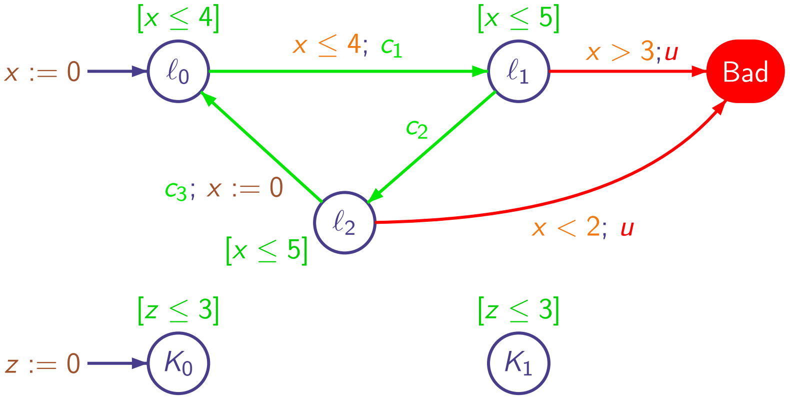

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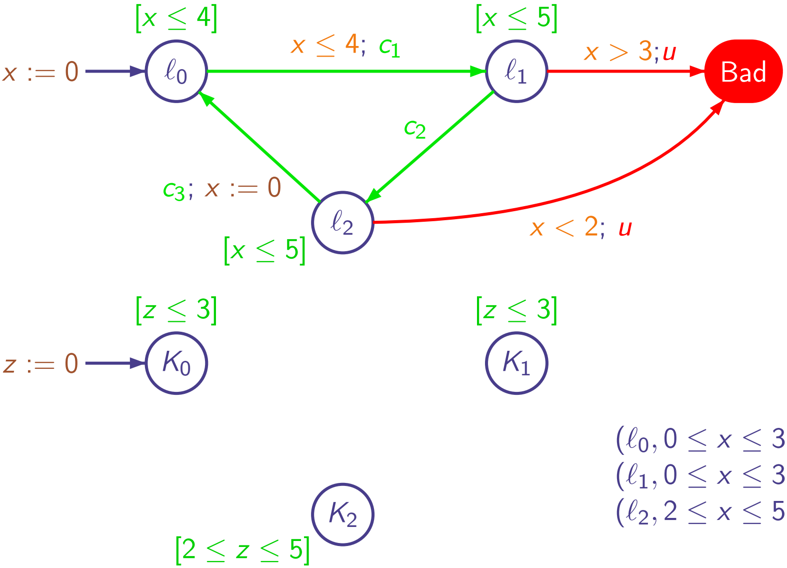

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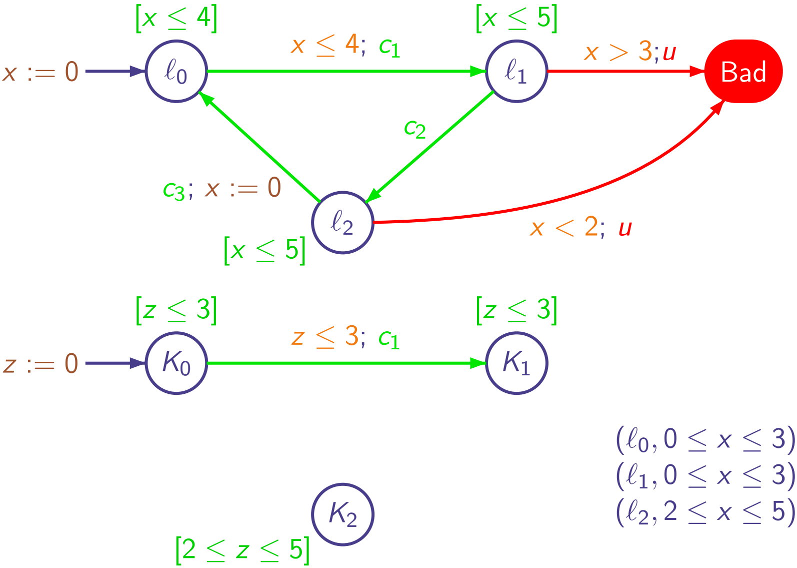

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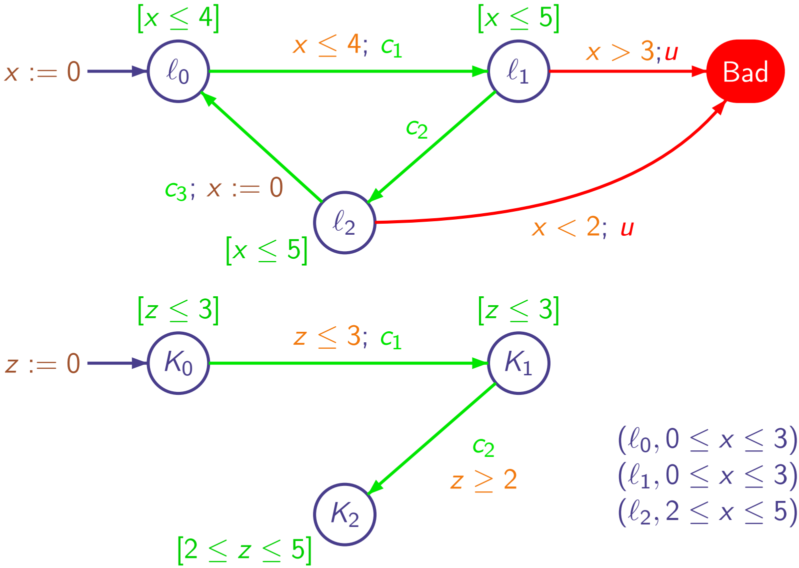

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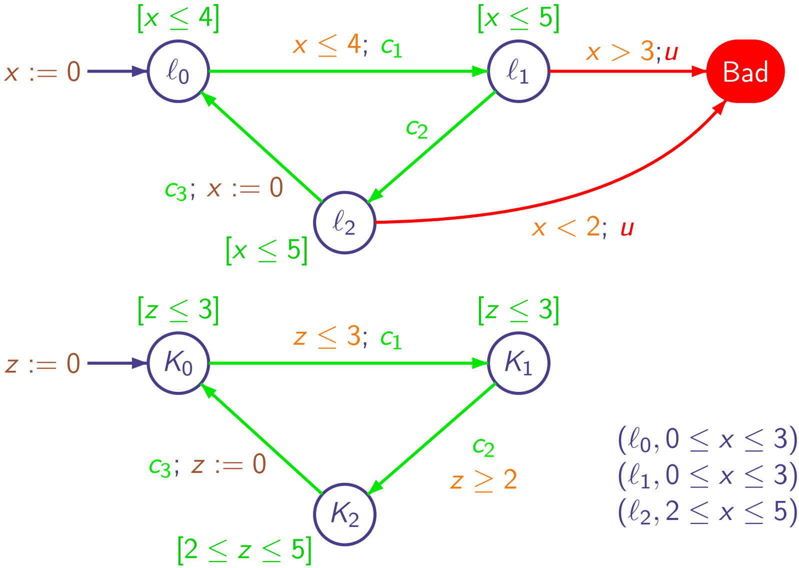
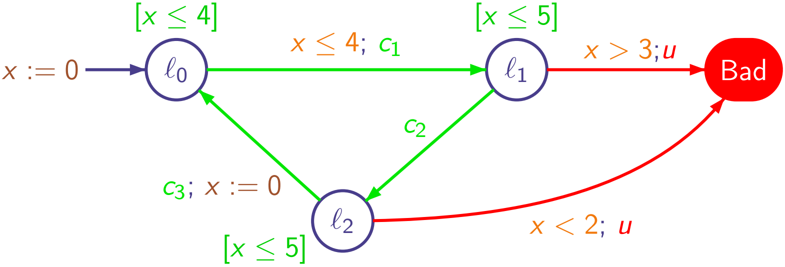
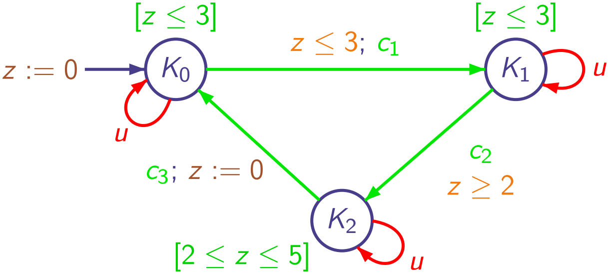


$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

Example of Computation





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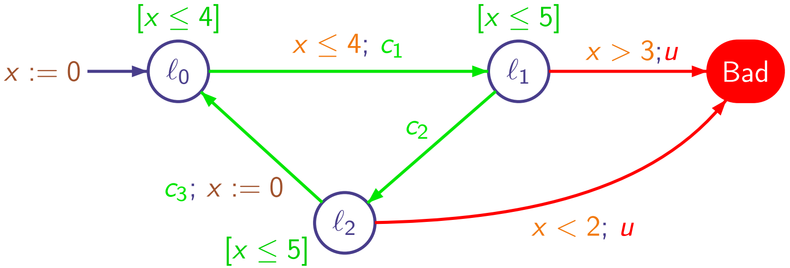
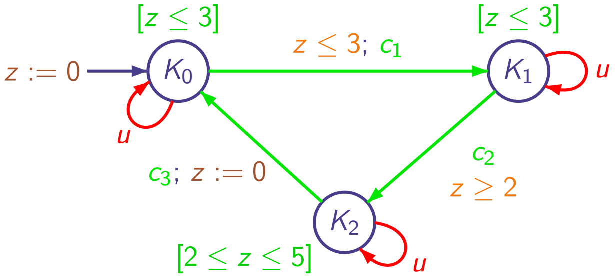
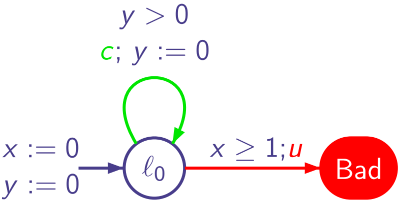




The Most Liberal Controller

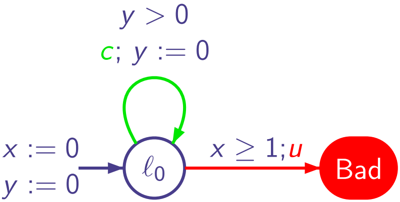
Problems with Dense-Time Control (I)



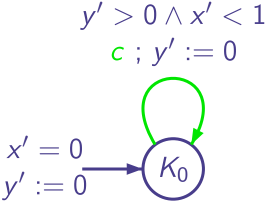
The System

The Controller is Zeno !!!

Problems with Dense-Time Control (I)



The System



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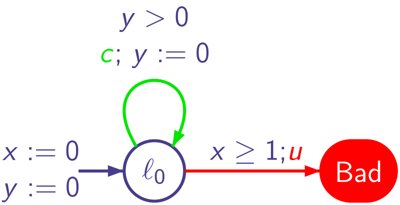

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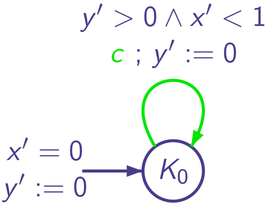

The Controller

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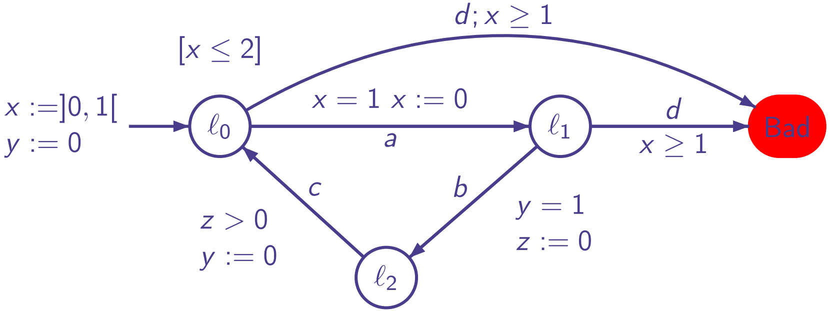
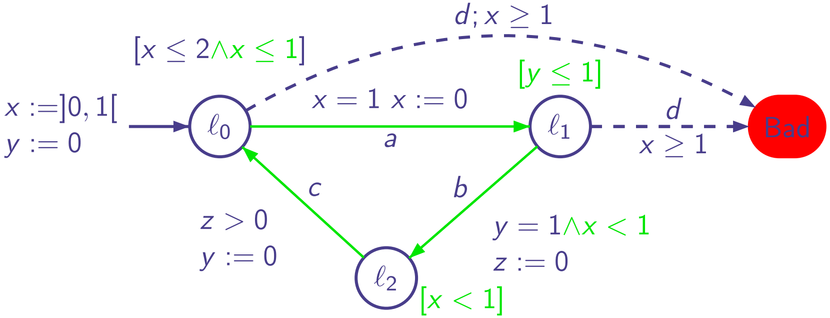
Problems with Dense-Time Control (II)



- Let δ_i : time spent in ℓ_2 on loop i
- The controller must ensure: $\sum_{i=0}^{i=+\infty} \delta_i < x_0 - y_0$

The Controller is Non-Zeno but not Implementable !!!

Problems with Dense-Time Control (II)



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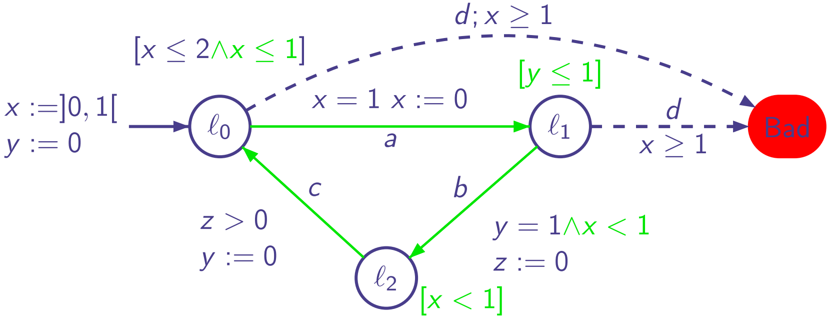

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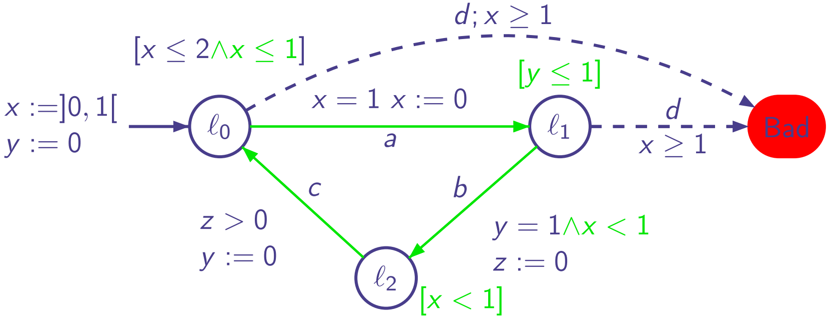

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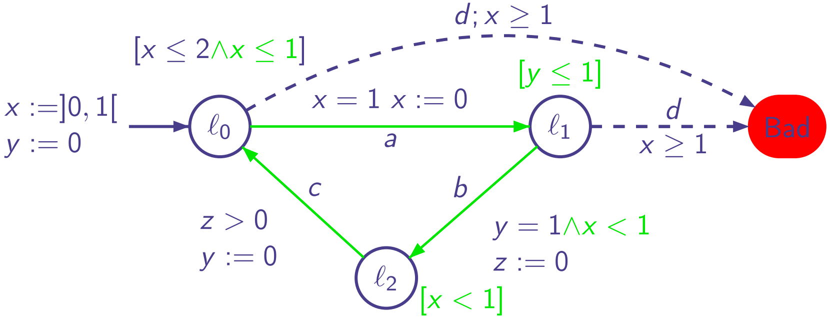

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Outline

- Verification & Control
- Control of Finite Automata
- Timed Game Automata
- Symbolic Algorithms for Timed Game Automata
- Conclusion

Partial Conclusion

Assumptions:

- Timed systems with full observation
- Ideal controller that operates in dense-time

Results:

- Control Problem is decidable for ω-regular objectives
- Control Synthesis Problem is effective
- Positional (or Memoryless) strategies are sufficient

Advanced Topics:

- Partial Observability
  Patricia
- Implementation
  Karine

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Timed Automata [Alur & Dill'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g,\,a,\,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in \mathrm{Act}$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Back

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ be a Timed Automaton.

A state (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a Timed Transition System $S_{\mathcal{A}} = (Q, q_0, \mathrm{Act} \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

$$\textbf{discrete transition: } (\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g\,,\,a\,,\,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models \mathrm{inv}(\ell') \end{cases}$$

$$\textbf{delay transition: } (\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models \mathrm{inv}(\ell)$$

Back

Definition (Outcome in Timed Games)

Let $G = (L, \ell_0, \mathrm{Act}, X, E, \mathrm{inv})$ be a TGA and f a strategy over G. The outcome $\mathrm{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is the subset of $\mathrm{Runs}((\ell, v), G)$ defined inductively by:

- $(\ell, v) \in \mathrm{Outcome}((\ell, v), f)$,
- if $\rho \in \mathrm{Outcome}((\ell, v), f)$ then $\rho' = \rho \xrightarrow{e} (\ell', v') \in \mathrm{Outcome}((\ell, v), f)$ if $\rho' \in \mathrm{Runs}((\ell, v), G)$ and one of the following three conditions hold:
  1. $e \in \mathrm{Act}_u$,
  2. $e \in \mathrm{Act}_c$ and $e = f(\rho)$,
  3. $e \in \mathbb{R}_{\geq 0}$ and $\forall 0 \leq e' < e, \exists (\ell'', v'') \in (L \times \mathbb{R}^X_{\geq 0})$ s.t. $last(\rho) \xrightarrow{e'} (\ell'', v'') \wedge f(\rho \xrightarrow{e'} (\ell'', v'')) = \lambda$.
- an infinite run ρ is in $\in \mathrm{Outcome}((\ell, v), f)$ if all the finite prefixes of ρ are in $\mathrm{Outcome}((\ell, v), f)$.