Sujet de stage de recherche / M2

Titre

Longest chains in Noetherian spaces.

Encadrants

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Description du sujet

Well structured transition systems (WSTS) form a large class of infinite state transition systems for which many properties of interest in verification are decidable. The prototypical example is coverability, which is decidable for all effective WSTS.

The complexity of these algorithms is high. Recent papers by Schmitz and coauthors have made decisive progress in the field [FFSS11, SS11, SS12, SS13, Sch14, LS15]. The key to those results is a finite understanding of maximal order types of well-quasi-orderings, a topic pioneered by Schmidt [Sch79], and their so-called controlled versions.

In another strand of reseach, Goubault-Larrecq generalized well-quasi-orders to so-called Noetherian topological spaces [Gou07, Gou10, Gou13], showing how this applied to verify an even larger class of infinite state transition systems, called topological WSTS.

The maximal order type of a well-quasi-ordered set happens to coincide with the ordinal length of the longest chain of upward closed subsets under inclusion. In Noetherian space theory, upward closed subsets must be replaced by open subsets. Since the coverability algorithms in WSTS and their topological analogues work by constructing increasing chains of open subsets, it is natural to study their ordinal length. It will then be natural to extend that to controlled versions of those lengths, in order to yield complexity bounds on the coverability algorithms.

Let us call the *height* of a Noetherian space the ordinal length of its longest chain of opens. The work should begin by :

– showing that the height of a coproduct of two Noetherian spaces is the natural sum of their heights ;
– showing the height of a product of two Noetherian spaces is the natural product of their heights ;
– determining the height of the space X^* of finite words under the word topology (generalizing the word embedding quasi-ordering) in terms of the height of X,

in increasing order of difficulty. The case of finite labeled trees would be an additional nice result.

This would be the first step to a PhD thesis on the subject, where one would expect the student to develop controlled versions of the above theorems, in order to extend the complexity bounds already obtained by Schmitz and coauthors in the case of ordinary WSTS.

Remarques

This internship requires some familiarity in the theories of well-quasi-orderings and ordinals, with a view toward verification. Basic knowledge of point-set topology, with a desire to learn more, will be appreciated. In any case, general mathematical proficiency is required.

Références

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[Gou10] Jean Goubault-Larrecq. Noetherian spaces in verification. In Samson Abramsky, Friedhelm Meyer auf der Heide, and Paul Spirakis, editors, *Proceedings of the 37th International Colloquium on Automata, Languages and Programming (ICALP'10) – Part II*, volume 6199 of *Lecture Notes in Computer Science*, pages 2–21, Bordeaux, France, July 2010. Springer.

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[LS15] Ranko Lazić and Sylvain Schmitz. Non-elementary complexities for branching VASS, MELL, and extensions. *ACM Transactions on Computational Logic*, 16(3 :20) :1–30, April 2015.

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